DATA MINING 2 Time Series – Matrix Profile, Motifs & Discords

Riccardo Guidotti

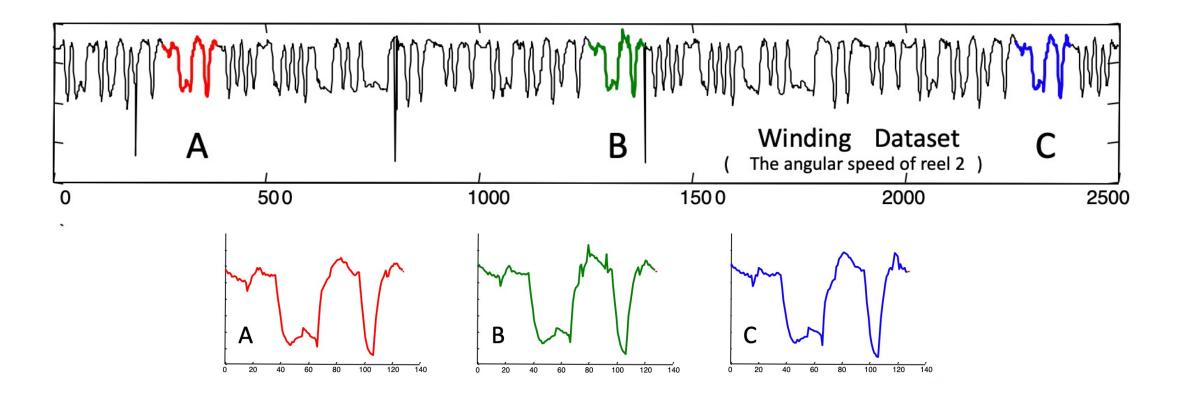
a.a. 2020/2021

Slides edited from Keogh Eamonn's tutorial



Time Series Motif Discovery

- Finding repeated patterns, i.e., pattern mining.
- Are there any repeated patterns, of length *m* in the TS?

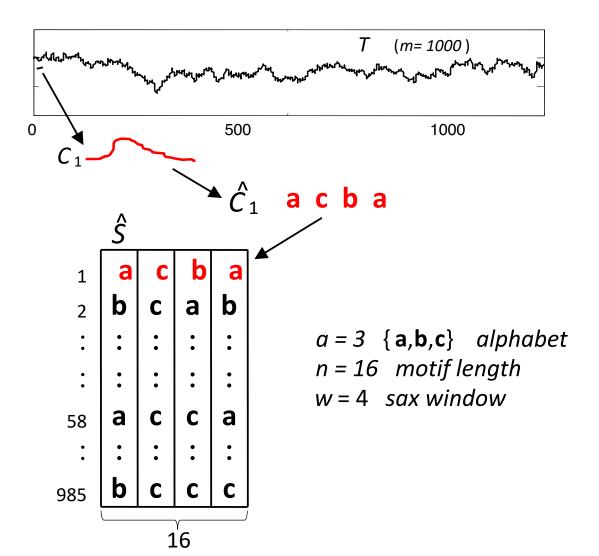


Why Finding Motifs?

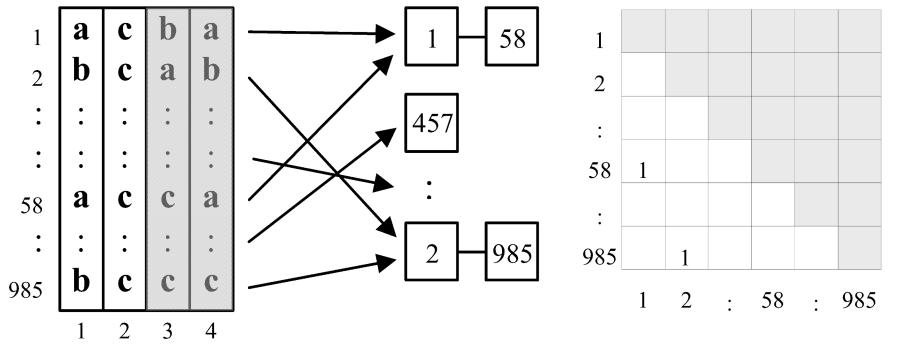
- Mining **association rules** in TS requires the discovery of motifs. These are referred to as primitive shapes and frequent patterns.
- Several **TS classifiers** work by constructing typical prototypes of each class. These prototypes may be considered motifs.
- Many **TS anomaly detection** algorithms consist of modeling normal behavior with a set of typical shapes (which we see as motifs), and detecting future patterns that are dissimilar to all typical shapes.

- Given a predefined motif length *m*, a brute-force method searches for motifs from all possible comparisons of subsequences.
- It is obviously very slow and computationally expensive.
- The most referenced algorithm is based on a hot idea from bioinformatics, random projection* and the fact that SAX allows use to lower bound discrete representations of TSs.
- J Buhler and M Tompa. Finding motifs using random projections. In RECOMB'01. 2001.

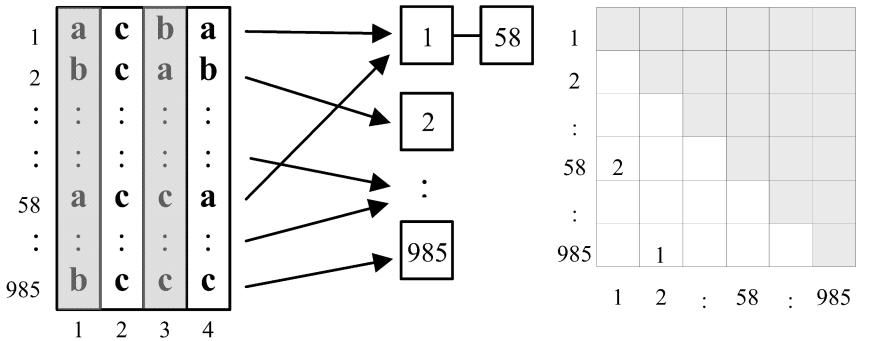
Assume that we have a time series T of length 1,000, and a motif of length 16, which occurs twice, at time T₁ and time T₅₈.



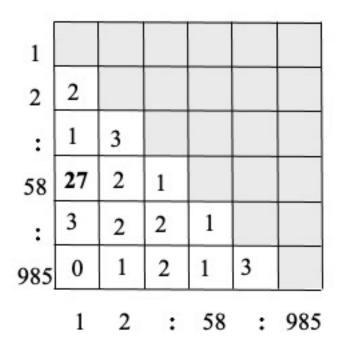
- A mask {1,2} was randomly chosen, so the values in columns {1,2} were used to project matrix into buckets.
- Collisions are recorded by incrementing the appropriate location in the collision matrix.



- A mask {2,4} was randomly chosen, so the values in columns {2,4} were used to project matrix into buckets.
- Once again, collisions are recorded by incrementing the appropriate location in the collision matrix.



- At the end of the random perturbations consider the motifs observing the matrix in decreasing order of occurrences.
- For instance this matrix indicates a high chance of having a motif staring at positions 1 and 58.
- The problem with this approach is that it is highly dependent from the approximation technique adopted.

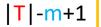


- The Matrix Profile (MP) is a data structure that annotates a TS and can be exploited for many purposed: e.g. efficient Motif Discovery.
- Given a time series, T and a desired subsequence length, m.



m

We can use sliding window of length *m* to extract all subsequences of length *m*.



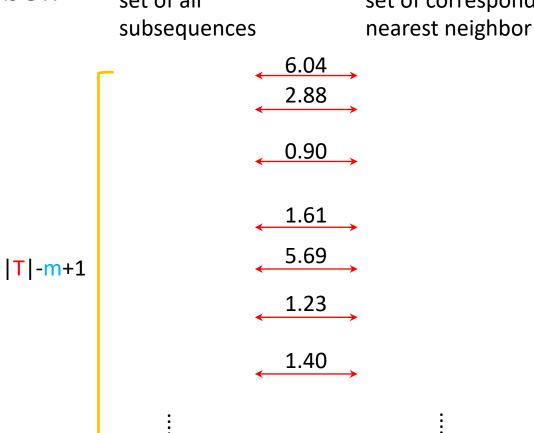
m

|T|-m+1

We can then compute the pairwise distance among these subsequences.

0	7.6952	7.7399	
7.6952	0	7.7106	
7.7399	7.7106	0	
	•••		

 For each subsequence we keep only the distance with the closest nearest neighbor. set of corresponding



• The distance to the corresponding nearest neighbor of each subsequence can be stored in a vector called **matrix profile P**.

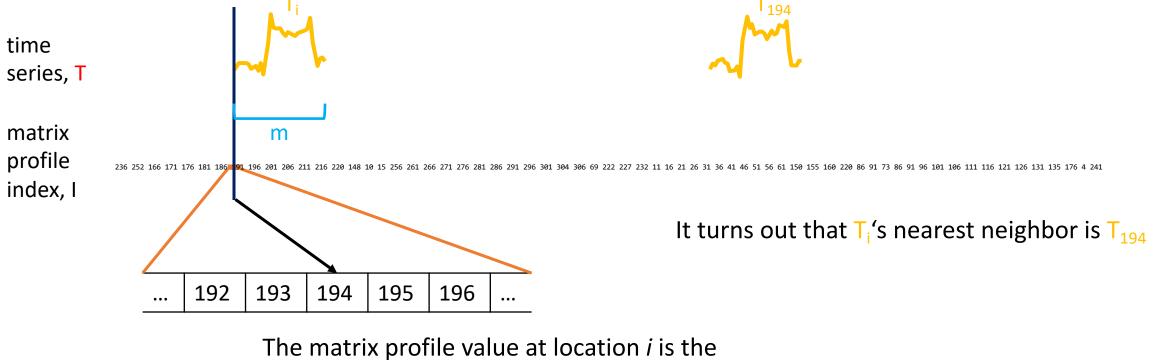
time series, T

matrix profile, P



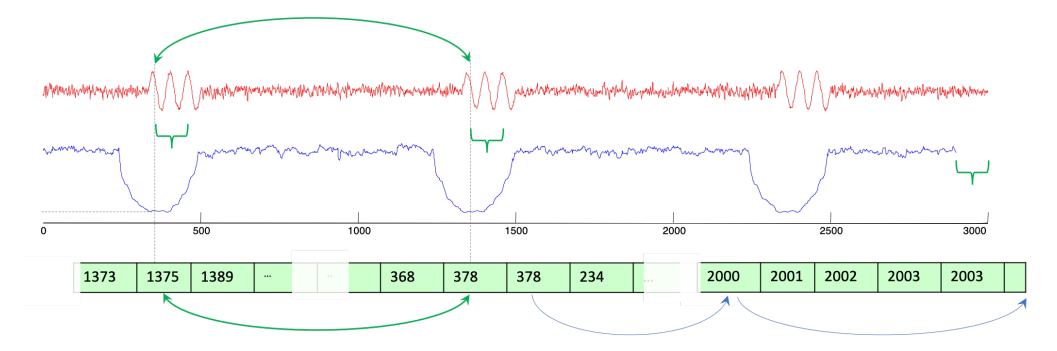
The matrix profile value at location i is the distance between T_i and its nearest neighbor

• The index of corresponding nearest neighbor of each subsequence is also stored in a vector called matrix profile index.



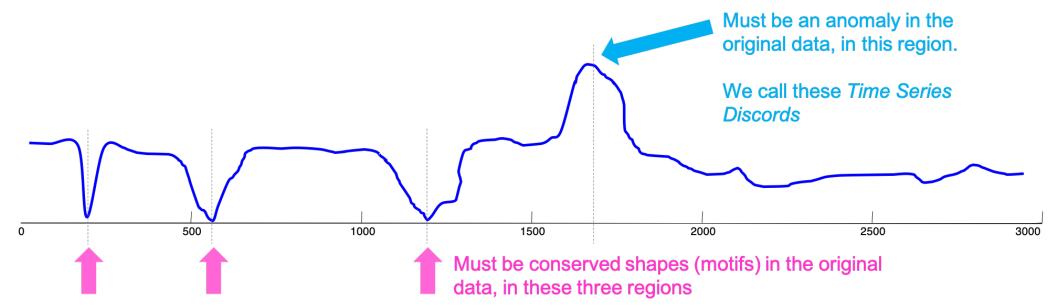
The matrix profile value at location i is the distance between T_i and its nearest neighbor

- The MP index allows to find the nearest neighbor to any subsequence in constant time.
- Note that the pointers in the matrix profile index are not necessarily symmetric.
- If A points to B, then B may or may not point to A
- The classic TS motif: the two smallest values in the MP must have the same value, and their pointers must be mutual.



How to "read" a Matrix Profile

- For relatively low values, you know that the subsequence in the original TS must have (at least one) relatively similar subsequence elsewhere in the data (such regions are "motifs")
- For relatively high values, you know that the subsequence in the original TS must be unique in its shape (such areas are anomalies).



• Given a time series, T and a desired subsequence length, m.



		inf																		
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Matrix profile is initialized as inf vector

This is just a toy example, so the values and the vector length does not fit the time series shown above

• Given a time series, T and a desired subsequence length, m.



At the first iteration, a subsequence T_i is randomly selected from T

• Given a time series, T and a desired subsequence length, m.



		inf																		
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We compute the distances between T_i and every subsequences from T (time complexity = O(|T|log(|T|))) We then put the distances in a vector based on the position of the subsequences

3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1

The distance between T_i and T_1 (first subsequence) is 3

• Given a time series, T and a desired subsequence length, m.

| inf |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

We compute the distances between T_i and every subsequences from T (time complexity = O(|T|log(|T|))) We them put the distances in a vector based on the position of the subsequences

Let say T_i happen to be the third subsequences, therefore the third value in the distance vector is 0

• Given a time series, T and a desired subsequence length, m.

m

	inf																		
min in mini mini mini mini mini mini mi																			
	3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1

• Given a time series, T and a desired subsequence length, m.

m

3	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
miı	n 🚺	Matrix profile is updated by apply elementwise minimum to these two vectors																
3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1

• Given a time series, T and a desired subsequence length, m.





3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1
5	2		5	5	-	5	-	2	5	0	T	2	5	-	0	0	2	-

After we finish update matrix profile for the first iteration

3	2	0 5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1]
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• Given a time series, T and a desired subsequence length, m.

 T_j



	3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1
1																			

In the second iteration, we randomly select another subsequence T_j and it happens to be the 12^{th} subsequences

• Given a time series, T and a desired subsequence length, m.

 $\int_{T_j}^{\infty}$

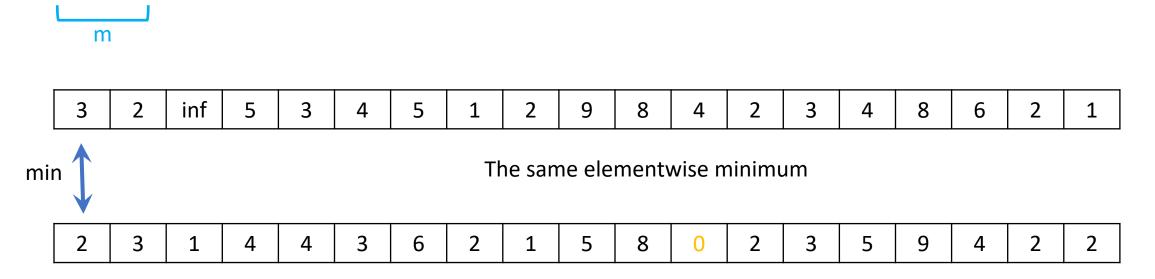


3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1

Once again, we compute the distance between T_i and every subsequences of T

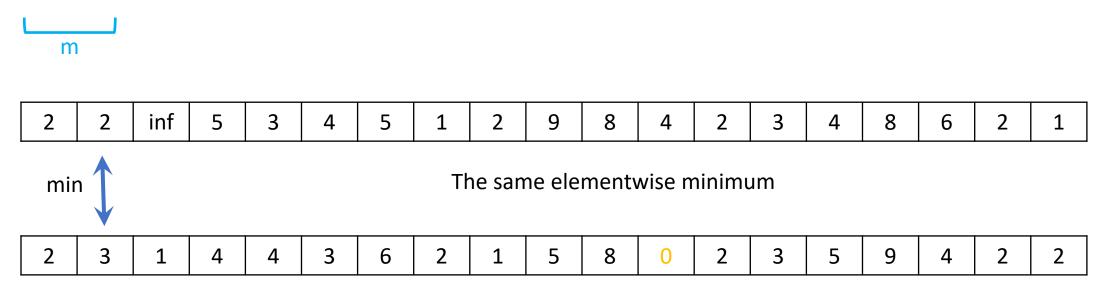
• Given a time series, T and a desired subsequence length, m.

Ti



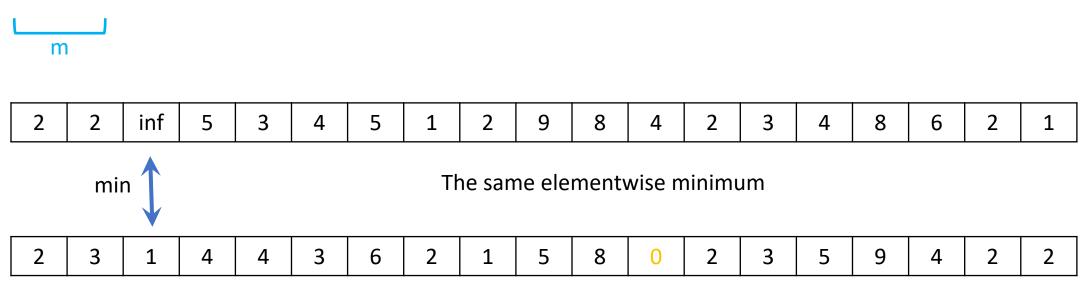
• Given a time series, T and a desired subsequence length, m.

Ti



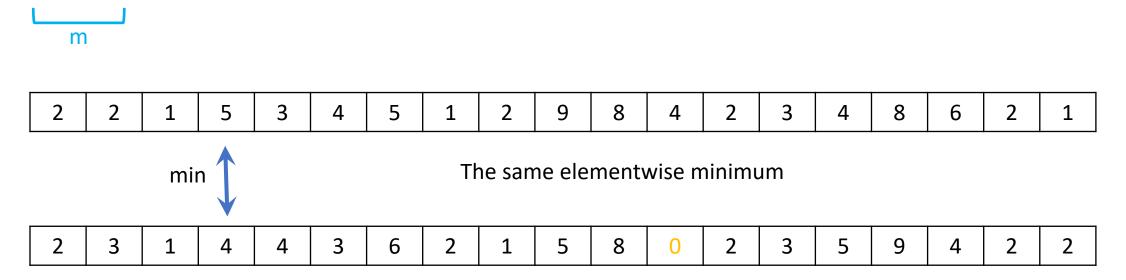
• Given a time series, T and a desired subsequence length, m.

 T_j

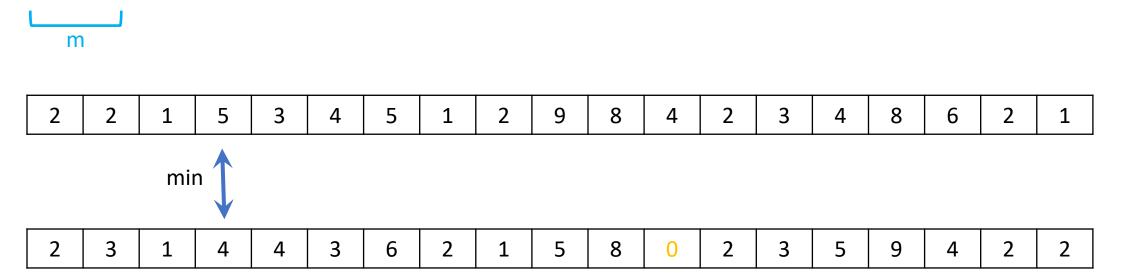


• Given a time series, T and a desired subsequence length, m.

 T_j



• Given a time series, T and a desired subsequence length, m.



We repeat the two steps (distance computation and update) until we have used every subsequences. The different indexes are analyzed in parallel and the distance is calculated using the Mueen's Algorithm for Similarity Search (MASS) https://www.cs.unm.edu/~mueen/FastestSimilaritySearch.html

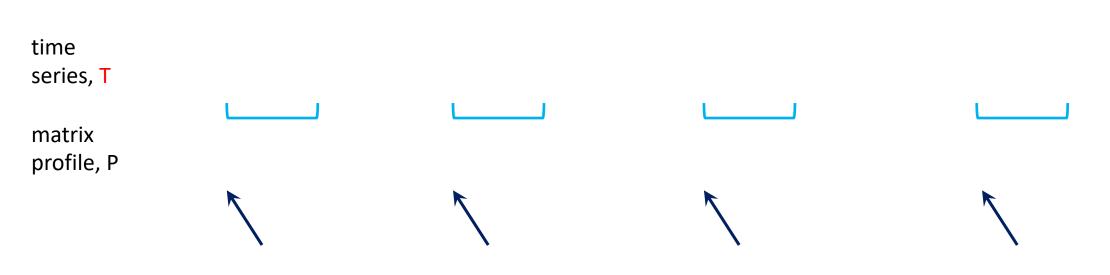
• Given a time series, T and a desired subsequence length, m.

m min

There are |T| subsequences and the distance computation is $O(|T|\log(|T|))$

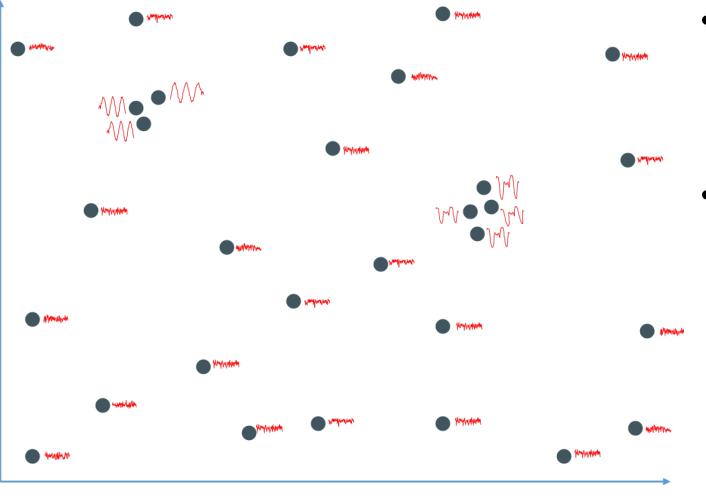
The overall time complexity is $O(|T|^2 \log(|T|))$

Motif Discovery From Matrix Profile



Local minimums are corresponding to motifs

Motif Discovery From Matrix Profile

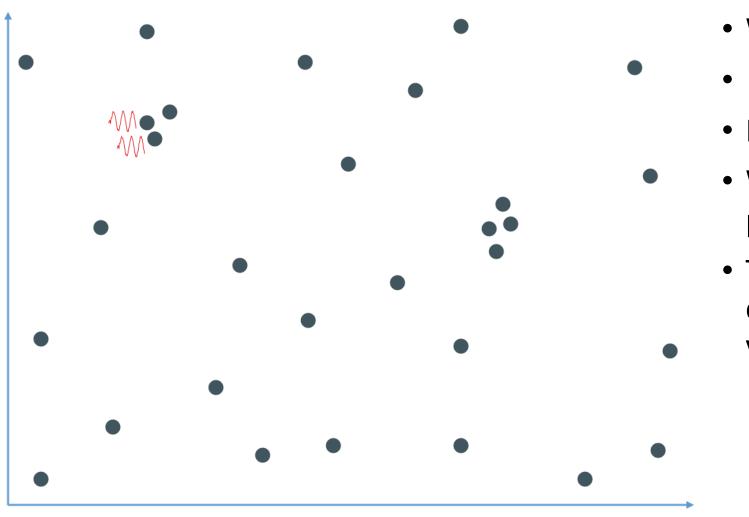


- It is sometime useful to think of time series subsequences as points in m-dimensional space.
 - In this view, dense regions in the m-dimensional space correspond to regions of the time series that have a low corresponding MP.

500

1000

Top-K Motifs



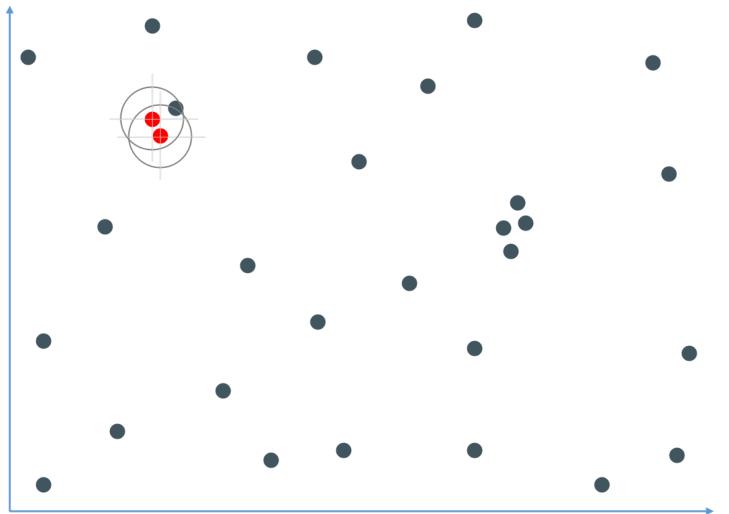
- We need a parameter R.
- 1 < R < (small number, say 3)
- Lets make R = 2 for now.

500

- We begin by finding the nearest pair of points, the *motif pair*....
 - This the pair of subsequences corresponding to lowest pair of values in the MP

1000

Top-K Motifs



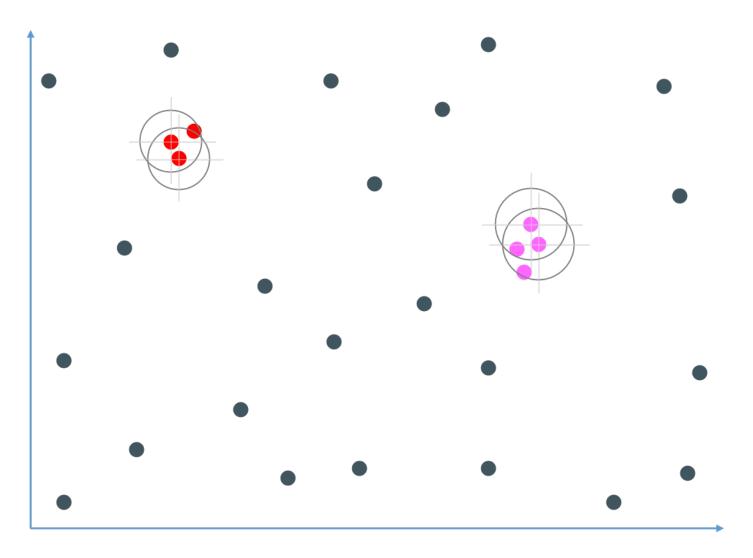
- We find the nearest pair of points are D1 apart.
- Lets draw a circle, D1 times R, around both points.
- Any points that are within either of these circles, are added to this motif, in this case just one.

1000

1500

 The Top-1 motif has three members, it is done.

Top-K Motifs

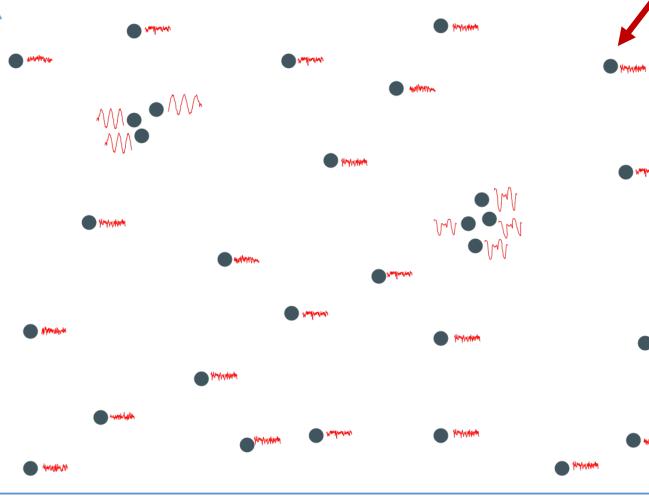


- Now lets find the Top-2 motif. We find the *nearest pair of points*, excluding anything from the top motif.
- The nearest pair of points are D2 apart.
- Lets draw a circle D2 times R, around both points.
- Any points that are within either of these circles, is added to this motif, in this case there are two for a total of four items in the Top-2 Motif

Top-K Motifs

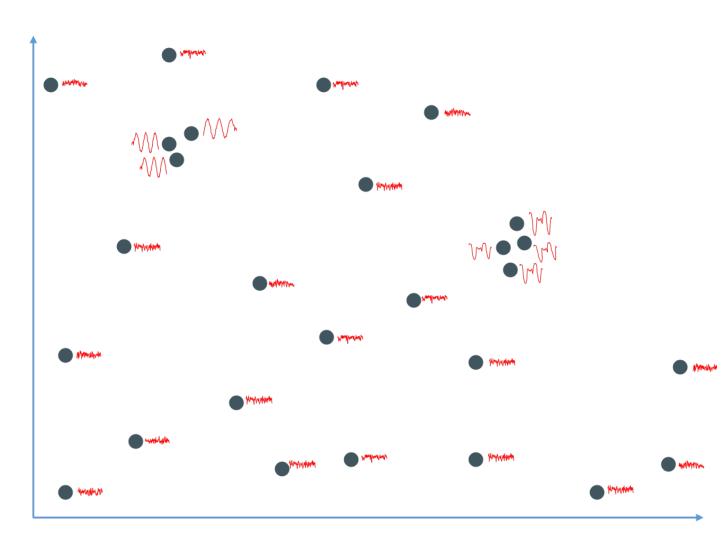
- We are done with the Top-2 Motif
- Note that we will always have:
 - $D_1 < D_2 < D_3 ... D_K$
- When to stop? (what is K?)
- We could use MDL or a predefined K.

Anomaly Discovery From Matrix Profile



- We need a parameter E of subsequences to exclude in the vicinity of the anomaly.
- Lets make E = 2 for now.
 - We begin by finding the subsequence with the highest distance in the MP
 - This corresponding to biggest anomaly

Top-K Anomaly



- Then we look for the E closest subsequences to the anomaly.
- We remove all of them.
- We can use a predefined K or the MDL to stop.

References

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Matrix Profile I: All Pairs Similarity Joins for Time Series A Unifying View that Includes Motifs, Discords and Shapelets

Chin-Chia Michael Yeh, Yan Zhu, Liudmila Ulanova, Nurjahan Begum, Yifei Ding Hoang Anh Dau, [†]Diego Furtado Silva, [†]Abdullah Mueen, and Eamonn Keogh University of California, Riverside, [†]Universidade de Sto Paulo, [†]University of New Mexico 1015, Janu00, J. hoge001, J. dug001, [†]Buor Ced, diegofabriagiscruc up le, zuween@una.eda, eamo

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Keywords-Time Series; Similarity Joins; Motif Discovery I. INTRODUCTION

The all-pairs-similarity-search (also known as similarit on) problem comes in several variants. The basic task is this: Given a collection of data objects, retrieve the nearest neighbor for each object. In the text domain the algorithm has such object. In the text domain the algorithm has actions in a host of problems, including community very, duplicate detection, collaborative filtering ring, and opery refinement [1]. While virtually all tex sing algorithms have analogues in time series dat g, there has been surprisingly little progress on Times subsequences All-Pairs-Similarity-Search (TSAPSS).

We believe that this lack of progress stems not from a lack interest in this useful primitive, but from the daunting nature of interest in this userial primitive, but from the datating nature of the problem. Consider the following example that reflects the needs of an industrial collaborator. A boiler at a chemica ressure once a minute. After a year, we have ime series of length 525,600. A plant manager may wish to do a similarity self-join on this data with week-long subsequences a similarly self-join on this statis with vect-long subsequences (10,000) to discover operating regardless (summer vs. winter or program of the self-long self-long self-long self-long self-algorithm, requires 12.280,092.900 Excitedan distance comparisons. If we assume each conclusion of this self-long self-sion with the class days. The core combinion of this work is to show that we can eradose this time to 6.3 shows, using an off-the-helf delakey comparer. Moreover, we show that this one can be compared and/or updated incrementally. Thus yes d maintain this join essentially forever on a standard

for most time series problems. While this may be considered good news, given the simplicity of implementing the nearest neighbor algorithm, there are some negative consequences of this. First, the nearest neighbor algorithm requires storing, and searching the entire dataset, resulting in a time and space complexity that limits in applicability, opeculably on resource-limited sensors. Second, beyond mere classification accuracy, we often wish to gain some imight num the data. mappin into the data. In this work we introduce a new time series primitive, file adopater, which addresses these limitations. Informally, so are time series subsequences which are in some sense an representative of a class. As we shall show with e empirical evaluations in diverse domains, algorithms base This is the author's version of an article published in Data Mining and authenticated version is available online at: https://doi.org/10.1007/s

ime series shapelet primitives can be interpretable, more accurs and significantly faster than state-of-the-art classifier Categories and Subject Descriptors ement]: Database Applications - Dat

General Terms

. INTRODUCTION

ABSTRACT

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Intredia

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In this work we present a novel time series data mining primitive called *inve zoriae* abspoker. Informally, happelet are time series subsequences which are in some scene maximally representative of a class. While we believe thappelets can have many uses in data mining, one dovious implication of them is to mitigate the two weaknesses of the neurost neighbor algorithm noted above.

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Such representations have been successful

warmo the subtle differences in the shapes

of the compared approaches and evaluated them on a univariate TS archive) and 12 multivariate time series datasets. By training 8,73 time series datasets, we propose the most exhaustive study of DNNs Keywords Deep learning \cdot Time series \cdot Classification \cdot Review

Deep learning for time series classificatio

Hassan Ismail Fawaz¹ · Germain Forestier^{1,2} · Jonathan W

Abstract Time Series Classification (TSC) is an important and chall

With the increase of time series data availability, hundreds of TSC a

Among these methods, only a few have considered Deep Neural Net task. This is surprising as deep learning has seen very successful appli have indeed revolutionized the field of computer vision especially w

architectures such as Residual and Convolutional Neural Networks. data such as text and audio can also be processed with DNNs to read

for document classification and speech recognition. In this article,

the-art performance of deep learning algorithms for TSC by preser

most recent DNN architectures for TSC. We give an overview of the

applications in various time series domains under a unified taxonor

provide an open source deep learning framework to the TSC communi

Lhassane Idoumghar¹ · Pierre-Alain Muller¹

1 Introduction

During the last two decades. Time Series Classification (TSC) has been considered as one of th most challenging problems in data mining (Yang and Wu, 2006; Esling and Agon, 2012). With the increase of temporal data availability (Silva et al., 2018), hundreds of TSC algorithms have been proposed since 2015 (Bagnall et al., 2017). Due to their natural temporal ordering, time series data e present in almost every task that requires some sort of human cognitive process (Längkvist et al., 2014). In fact, any classification problem, using data that is registered taking into account some notion of ordering, can be cast as a TSC problem (Cristian Borges Gamboa, 2017). Time series are encountered in many real-world applications ranging from electronic health records (Rajkomar et al., 2018) and human activity recognition (Nweke et al., 2018; Wang et al., 2018) to acoustic scene classification (Nwe et al., 2017) and cyber-security (Susto et al., 2018). In addition, the diversity of the datasets' types in the UCR/UEA archive (Chen et al., 2015); Bagnall et al., 2017) (the largest repository of time series datasets) shows the different applications of the TSC problem

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Time Series Shapelets: A New Primitive for Data Mining



• While our exact algorithm is extremely scalable, for extremely large distancts we can compute the results in an anytime fishion, allowing ultra-fast approximate solutions. Having computed the similarity join for a dataset, we can incrementally update it very efficiently. In many domains this means we can effectively maintime exact joins on the similar data forever. Or method provided and data which us we call shows, in a new intervent of the similarity of the second to new intervent of the similarity of the second to mark the similarity of the similarity of the second to mark the similarity of the similarity of the second new intervent of the similarity of the similarity of the new impossible task in this domain.

Our algorithm is embarrassingly parallelizable, both on multicore processors and in distributed existence

Lexiang Ye

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Because we are defining and solving a new problem, we will ta some time to consider a detailed motivating example. Figure Classification of time series has been attracting great interest over the past decade. Recent empirical evidence has strongly suggested that the simple nearest neighbor algorithm is very difficult to beat for most time series problems. While this may be considered good some time to consider a detailed motivating example. Figure 3 shows some examples of leaves from two classes, Uritor dioic (stinging nettles) and Verbena writefolia. These two plants an commonly confused, hence the colloquial name "false nettle" fo Verbena uricifolia.

Eamonn Keogh

Figure 1: Samples of leaves from two species. Note that severa leaves have the insect-bite damage appose we wish to build a classifier to distinguish thes

plants; what features should we use? Since the intra-variability o color and size within each class completely dwarfs the inte color and size within each class completely dwarfs the intra-vinsibility between classes, our best hope is based on the shapes of the larces. However, as we can see in Figure 1, the differences in the global shape are very worked. Furthermore, it is very common for leves to have distortions or "occlusion" due to insect damage, and these are lakely to confine any global measures of shape. Instead we attempt the following. We first convert each left into a one-dimensional representation as shown



Figure 2: A shape can be converted into a one series" representation. The reason for the highlig time series will be made apparent shortly

ation, clustering and outlier detection of share Classification, clustering and outlier detection or snapes in recent years [8]. However, here we find that using a nearest neighbor classifier with either the (rotation invariant) Euclidean distance on Dynamic Time Warping (DTW) distance does not significantly outperform random guessing. The reason for the poo performance of these otherwise very competitive classifiers seem bites, and different stem lengths), and this noise is enough to

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Time Series Analysis and Its Applications

With R Examples

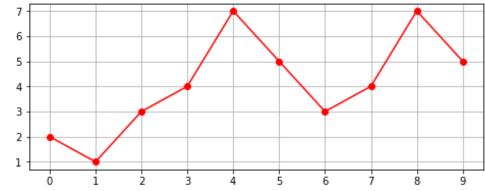
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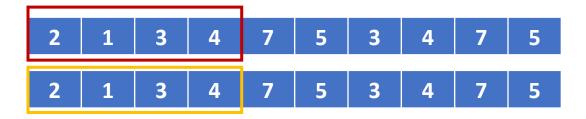
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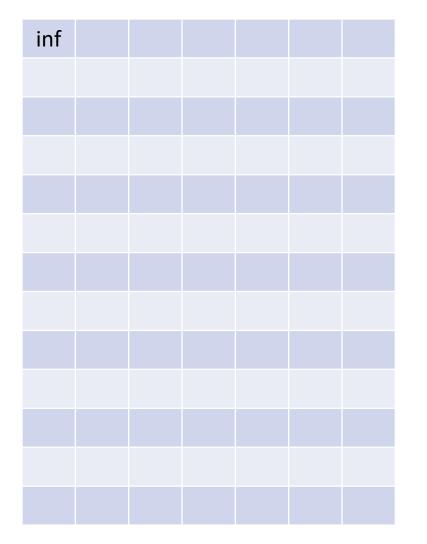
Exercises Matrix Profile

Given the TS x = <2,1,3,4,7,5,3,4,7,5>

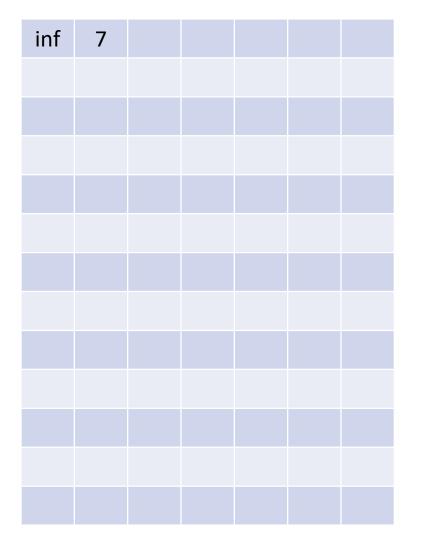
- 1. Build the Matrix Profile for x with m=4 using the Manhattan distance as distance function between subsequences.
- 2. Draw the Matrix Profile
- 3. Identify the motifs with distance equals 0 and length equals to m
- 4. Which is a correct value for m that would have retrieved less motifs with distance equals to 0?













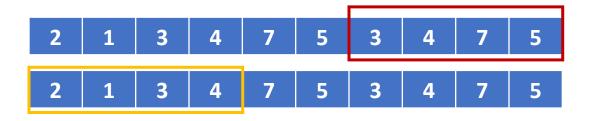
inf	7	9		



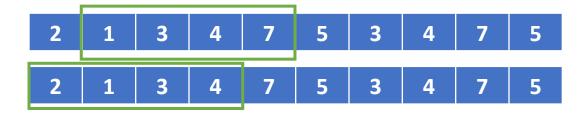
inf	7	9	11		

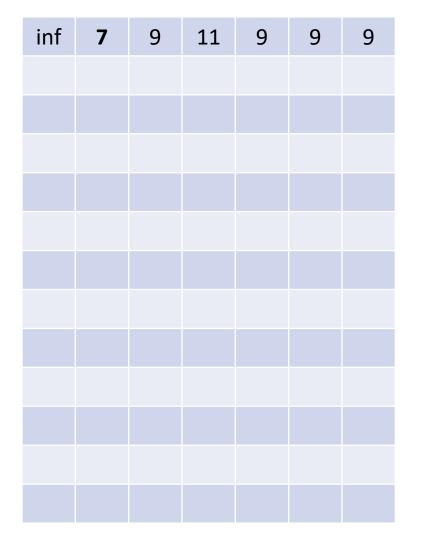


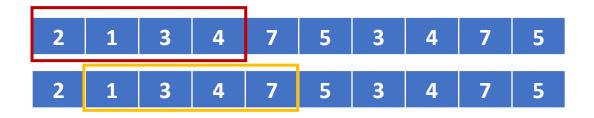
inf	7	9	11	9	



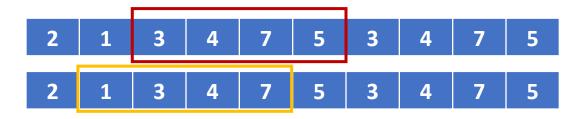
inf	7	9	11	9	9	9







inf	7	9	11	9	9	9
7						



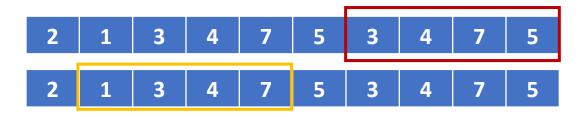
inf	7	9	11	9	9	9
7	inf	8				



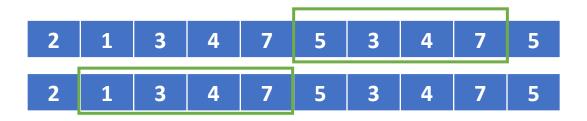
inf	7	9	11	9	9	9
7	inf	8	12			



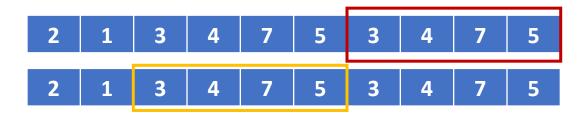
inf	7	9	11	9	9	9
7	inf	8	12	12		



inf	7	9	11	9	9	9
7	inf	8	12	12	4	8

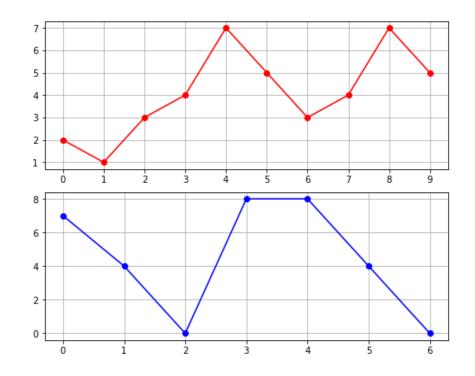


inf	7	9	11	9	9	9
7	inf	8	12	12	4	8



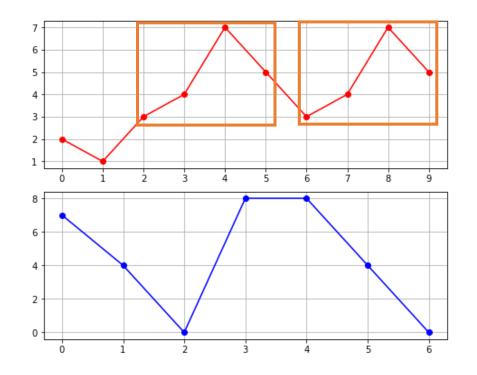
inf	7	9	11	9	9	9
7	inf	8	12	12	4	8
9	10	inf	8	9	8	0

- x = <2, 1, 3, 4, 7, 5, 3, 4, 7, 5>
- mp = < 7, 4, 0, 8, 8, 4, 0 >



• x = <2, 1, 3, 4, 7, 5, 3, 4, 7, 5 >

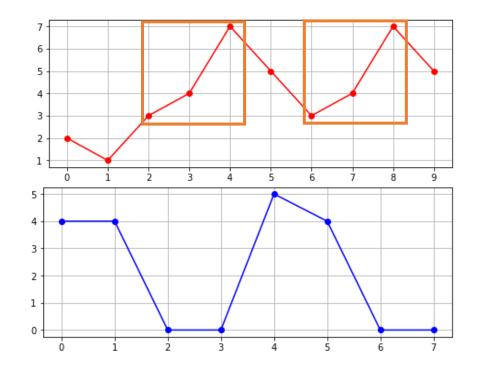
• mp = < 7, 4, 0, 8, 8, 4, 0 >



m=4

• x = <2, 1, 3, 4, 7, 5, 3, 4, 7, 5 >

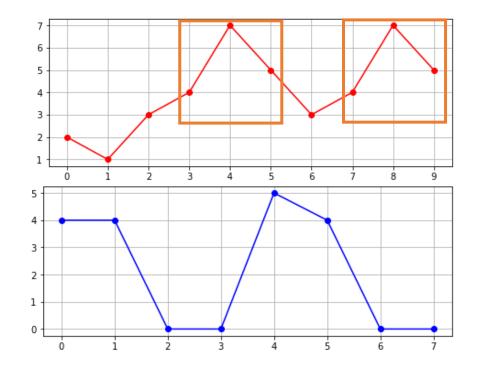
• mp = < 7, 4, 0, 8, 8, 4, 0 >



m=3

• x = <2, 1, 3, 4, 7, 5, 3, 4, 7, 5 >

• mp = < 7, 4, 0, 8, 8, 4, 0 >



m=3

Given the TS x = <5,5,3,5,5,1>

- 1. Build the Matrix Profile for x with m=2 using the Manahttan distance as distance function between subsequences.
- 2. Draw the Matrix Profile
- 3. Identify the motifs with distance equals 0 and length equals to m

