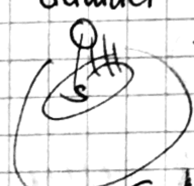
define $N_k(u)$

(triangle inequality implicitly used)

$k = \sqrt{n}$

$\alpha > 2$

[1] S = sample $|S| = \alpha \frac{n}{k} \ln n$ nodes unif. at random

[2] w = farthest node from S (dummy  + BFS(dummy))

[3] return $\tilde{D} = \max BFS(v)$ for $v \in S \cup \underbrace{N_k(w)}_{\text{stop BFS}(w) \text{ after } k \text{ nodes}}$

Prop S is hitting set for $N_k(u)$, $u \in V$ w.h.p.

proof

$$p = Pr\left(S \cap N_k(u) = \emptyset\right) = \left(\frac{\overbrace{n-k}^{\#\{x \notin N_k(u)\}}}{n}\right)^{\overbrace{|S|}^{\#\text{experiments}}} = \underbrace{\left(1 - \frac{k}{n}\right)}_{\sim e^{-k/n}}^{|S|} =$$

$$\cong e^{-\frac{k|S|}{n}} = e^{-\frac{k\left(\alpha \frac{n}{k} \ln n\right)}{n}} = n^{-\alpha}$$

Union bound: $n \cdot n^{-\alpha} = \frac{1}{n^{\alpha-1}}$ at least one $N_k(u)$ is not hit

over all $u \in V$ for $N_k(u)$

$\Rightarrow$ prob. $\geq \underbrace{1 - \frac{1}{n^{\alpha-1}}}_{\text{w.h.p. } \alpha > 2}$ that S is hitting set ☒

Prop Let $D = 3h + z$ where $z \in \{0,1,2\}$

w.h.p. $2h + z \leq \tilde{D} \leq D$ $z = 0,1$

$2h+1$ $z = 3$

proof Let a,b s.t. $D = d(a,b)$ diametral nodes

(a) $d(w,S) \leq h \Rightarrow d(a,S) \leq h \Rightarrow BFS(x) \geq D - h = 2h+z$

where $x \in S$ is the closest point to a

ow diameter $> D$

(b) $d(w,S) > h$ (i.e. every node in S is $> h$ far from w)

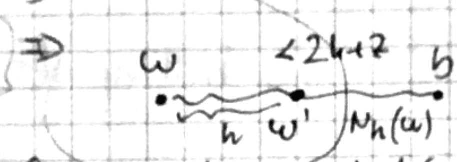
(b.1) $BFS(w) \geq 2h+2 \Rightarrow$ DONE

(b.2) $BFS(w) < 2h+2 \Rightarrow d(w,b) < 2h+2$

S hits $N_k(w)$ w.h.p. $\Rightarrow$

w.h.p. $\exists\, x \in N_k(w) \cap S$ s.t. $d(w,x) > h \Rightarrow$

$\{N_k(w)$ contains at least all nodes at distance h from $w\}$ $\Rightarrow$



w.h.p. $\exists\, w' \in N_k(w)$ on the min path from w to b s.t. $d(w,w') = h$

$\Rightarrow d(w',b) < h+2 \Rightarrow d(w',a) \geq D - (h+2-1)$
$= 2h+1$

$\Rightarrow BFS(w') \geq 2h+1$ □