Models of computation (MOD) 2013/14
Mid-term exam – April 13, 2015

[Ex. 1]
Consider the IMP command

$$w \stackrel{\text{def}}{=} \textbf{while } y > 0 \textbf{ do } (r := r \times x \;;\; y := y - 1)$$

1. Let $c \stackrel{\text{def}}{=} (r := 1 \;;\; w)$ and $\sigma \stackrel{\text{def}}{=} [9/x, 2/y]$. Use goal-oriented derivation, according to the operational semantics of IMP, to find the memory σ' such that $\langle c, \sigma \rangle \to \sigma'$, if it exists.
2. (**difficult**) Compute the denotational semantics $\mathscr{C}[\![w]\!] = \text{fix}\ \Gamma$.
   Hint: Prove that letting $\varphi_n \stackrel{\text{def}}{=} \Gamma^n \bot_{\Sigma \to \Sigma_\bot}$ it holds $\forall n \geq 1$

   $$\varphi_n = \lambda\sigma.\ (\sigma y > 0) \to (\ (\sigma y \geq n) \to \bot_{\Sigma_\bot}\ ,\ \sigma[\sigma r \times (\sigma x)^{\sigma y}/r, 0/y]\ )\ ,\ \sigma$$

[Ex. 2]
Let $D = \omega \cup \{\infty_0, \infty_1\}$ and $\sqsubseteq$ be the relation over D such that:

- for any pair of natural numbers $n, m \in \omega$, we let $n \sqsubseteq m$ iff $n \leq m$;
- for any natural number $n \in \omega$, we let $n \sqsubseteq \infty_0$ iff n is even;
- for any natural number $n \in \omega$, we let $n \sqsubseteq \infty_1$ iff n is odd;
- and we set $\infty_0 \sqsubseteq \infty_0 \sqsubseteq \infty_1 \sqsubseteq \infty_1$.

Is $(D, \sqsubseteq)$ a CPO? Explain.

[Ex. 3]
Let x, y, w be variables of type int, and $f : int \to (int \to int)$. Consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec}\, f.\ \lambda\, x.\ \textbf{if } x \textbf{ then } (\lambda\, y.\ (y + w)) \textbf{ else } (f\ w)$$

1. Compute the term $t[(f\ x\ y)/w]$.
2. Compute the term $t[(f\ x\ y)/x]$.

Hint: You are allowed to introduce additional (typed) variables if needed.

[Ex. 4]
Is it possible to assign a type to the HOFL pre-term below? If yes, compute its principal type.

$$\textbf{rec}\, f.\ \lambda\, x.\ (\ f\ \textbf{fst}(x)\ ,\ f\ \textbf{snd}(x)\ )$$