# Models of computation (MOD) 2015/16

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**[Ex. 1]** Let $var : (Aexp \cup Bexp) \to \wp(\mathbf{Loc})$ and $asgn : Com \to \wp(\mathbf{Loc})$ be the functions that return, respectively, the set of variables appearing in an expression and the set of variables appearing in the left-hand side of some assignment within a command.

1. Define *var* and prove that the predicates below hold (for all $a, b, \sigma, n, v$):

$$P(\ \langle a, \sigma\rangle \to n\ ) \stackrel{\text{def}}{=} \forall \sigma'.\ (\forall y \in var(a).\ \sigma(y) = \sigma'(y)) \ \Rightarrow\ \langle a, \sigma'\rangle \to n$$
$$Q(\ \langle b, \sigma\rangle \to v\ ) \stackrel{\text{def}}{=} \forall \sigma'.\ (\forall y \in var(b).\ \sigma(y) = \sigma'(y)) \ \Rightarrow\ \langle b, \sigma'\rangle \to v$$

   You can safely skip the proofs of cases analogous to ones already shown.

2. Define *asgn* and prove that the following predicate holds (for all $c, \sigma, \sigma'$):

$$R(\ \langle c, \sigma\rangle \to \sigma'\ ) \stackrel{\text{def}}{=} \forall y \notin asgn(c).\ \sigma(y) = \sigma'(y)$$

3. Let $b \in Bexp$ and $c \in Com$ such that $var(b) \cap asgn(c) = \emptyset$.
   Exploit the above properties and the the rule for divergence to prove that $\forall \sigma.\ (\langle b, \sigma\rangle \to \mathbf{true} \ \Rightarrow\ \langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma\rangle \not\to)$.

**[Ex. 2]** A relation $R \subseteq X \times X$ over a set $X$ is *transitive* if for all $x, y, z \in X$:

$$(\ (x, y) \in R\ \wedge\ (y, z) \in R\ ) \ \Rightarrow\ (x, z) \in R.$$

Let $\mathbf{TR} \subseteq \wp(\mathbb{N} \times \mathbb{N})$ be the set of transitive relations over natural numbers. Obviously, $(\mathbf{TR}, \subseteq)$ is a partial order whose bottom element is $\emptyset$.

1. Prove that $(\mathbf{TR}, \subseteq)$ is complete.

2. Consider the function $f : (\mathbf{TR}, \subseteq) \to (\wp(\mathbb{N}), \subseteq)$ defined by letting

$$\forall R \in \mathbf{TR}.\ f(R) \stackrel{\text{def}}{=} \{\, x \mid \exists y.\ (x, y) \in R \vee (y, x) \in R \,\}$$

   Prove that $f$ is monotone and continuous.

**[Ex. 3]** Modify the operational semantics of the conditional construct of HOFL so that **if** $t$ **then** $t_0$ **else** $t_1$ has a canonical form if and only if all terms $t, t_0, t_1$ have canonical forms. Then define the recursive HOFL term *fact* for computing the factorial and explain the reason why the term (*fact* 0) has no canonical form according to the new operational semantics.

**[Ex. 4]** Let us consider the fragment of HM-logic given by the formulas:

$$\phi \ ::=\ true \ \mid\ \Diamond_{\mu}\phi$$

Prove that trace equivalence coincide with the logical equivalence induced by the above fragment of HM-logic.