Models of computation (MOD) 2016/17
Exam – June 15, 2017

[Ex. 1] (1st mid-term / regular exam)
Let IMP* be the variant of IMP where the **while-do** construct is replaced by the construct c^*, whose operational semantics is defined by the rules

$$\frac{}{\langle c^*, \sigma\rangle \to \sigma} \qquad \frac{\langle c, \sigma\rangle \to \sigma'' \quad \langle c^*, \sigma''\rangle \to \sigma'}{\langle c^*, \sigma\rangle \to \sigma'}$$

1. Let $P(c) \stackrel{\text{def}}{=} \forall \sigma.\ \exists \sigma'.\ \langle c, \sigma\rangle \to \sigma'$. Prove that the predicate $P(c)$ holds for any command c of IMP*.
2. Give a command c of IMP* and a state σ such that there are infinitely many different σ' with $\langle c^*, \sigma\rangle \to \sigma'$.
3. Define the notion of operational equivalence and show that the command $x := 0$ and $(x := 0)^*$ are not operationally equivalent.

[Ex. 2] (1st mid-term / regular exam)
A *down-set* of a partial order $(P, \sqsubseteq_P)$ is a set $X \subseteq P$ such that

$$\forall x \in X.\ \forall p \in P.\ p \sqsubseteq_P x \Rightarrow p \in X.$$

Given any two partial orders $(P, \sqsubseteq_P)$ and $(Q, \sqsubseteq_Q)$ and a monotone function $f : P \to Q$ prove that if Y is a down-set of $(Q, \sqsubseteq_Q)$ then its counterimage

$$f^{-1}(Y) = \{x \in P \mid f(x) \in Y\}$$

is a down-set of $(P, \sqsubseteq_P)$.

[Ex. 3] (1st mid-term only)
Prove that the statement in Exercise 2 can be reversed. Namely, given any two partial orders $(P, \sqsubseteq_P)$ and $(Q, \sqsubseteq_Q)$ and a function $f : P \to Q$ prove that: if the counterimage $f^{-1}(Y)$ of any down-set Y of $(Q, \sqsubseteq_Q)$ is itself a down-set of $(P, \sqsubseteq_P)$, then f is monotone.

[Ex. 4] (2nd mid-term / regular exam)
Prove that for any variable x, any environment ρ any term t and any closed term t_0 we have that

$$[\![(\lambda x.\ t)\ t_0]\!]\,\rho = [\![t[^{t_0}/_x]]\!]\,\rho$$

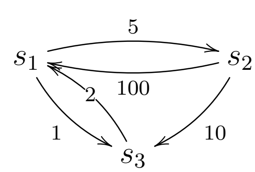
according to the lazy denotational semantics of HOFL. Justify each passage of the proof.

[Ex. 5] (2nd mid-term / regular exam)
Given a renaming ϕ, consider the CCS processes

$$p \stackrel{\text{def}}{=} \textbf{rec}\ x.\ ((\alpha.x)[\phi]) \qquad q \stackrel{\text{def}}{=} (\textbf{rec}\ x.\ \alpha.x)[\phi]$$

1. Draw the LTSs (at least in part) of p and q.
2. Define a concrete renaming ϕ such that $p \not\simeq q$ (i.e., p and q are not strong bisimilar).
3. Under which hypothesis on ϕ are p and q strong bisimilar?

[Ex. 6] (2nd mid-term only)
Suppose a machine lifecycle alternates between states s_1 (working), s_2 (malfunction) and s_3 (on repair), as modeled by the CTMC below.



1. Write the infinitesimal generator matrix and the embedded DTMC.
2. What is the probability to leave the state s_1 within t units of time?
3. What is the probability to find the machine working after many units of time?