An Introduction to Mixed Integer Linear Problems (MILP)

(Ragsdale : Chap. 6)

Many business problems need integer solutions (e.g. how many employees to assign to each shift, how many vehicles to purchase...)

→ (Mixed) Integer Linear Programming : certain decision variables must assume only integer values

An example : Blue Ridge Hot Tubs

$$\text{Max } 350\,x_1 + 300\,x_2$$

$$x_1 + x_2 \leq 200$$

$$9\,x_1 + 6\,x_2 \leq 1566 \quad 1520$$

$$12\,x_1 + 16\,x_2 \leq 2880 \quad 2650$$

$$x_1,\ x_2 \geq 0$$

integrality constraints $\quad x_1,\ x_2$ integer

- The optimal solution was integer ($x_1^* = 122$, $x_2^* = 78$)

- However, by changing the two LHS we would get $x_1^* = 116,9444$ and $x_2^* = 77,9167$: so, integrality constraints have to be added to the model (from LP to (M)ILP)

How can we address ILP ?

1) Solving its <u>Linear Relaxation</u>, i.e. eliminating the integrality constraints, and then rounding the obtained solution

e.g. $\tilde{x}_1 = 116$ $\hat{x}_2 = 77$ (profit: 63.700)

<u>Obs</u>: this is not necessarily an optimum solution to the ILP

<u>Obs</u>: the optimum LP value, 64.306, is <u>an upper bound</u> to the optimum ILP value (for a maximization problem)

2) Solving directly the ILP model (e.g. via the EXCEL solver)

< Read Section 6.7 to implement the Integer Blue Ridge Hot Tubs as a spreadsheet model: EXCEL file available >

Other examples

1) A fixed-charge problem
(Ragsdale : Sect. 6.12 , 6.13)

- Remington Manufacturing is planning its next production cycle
- 3 products (say Product 1, Product 2 and Product 3), each of which must undergo machining, grinding and assembly operations to be completed