I. L. P. model : (p-median)

$$\text{Min} \sum_{i \in I} \sum_{j \in J} h_i \cdot d_{ij} \cdot y_{ij}$$

Subject to :

$$\sum_{j \in J} x_j = p$$

$$\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I$$

$$y_{ij} \leq x_j \quad \forall i \in I, j \in J$$

$$x_j \in \{0, 1\} \quad \forall j \in J$$

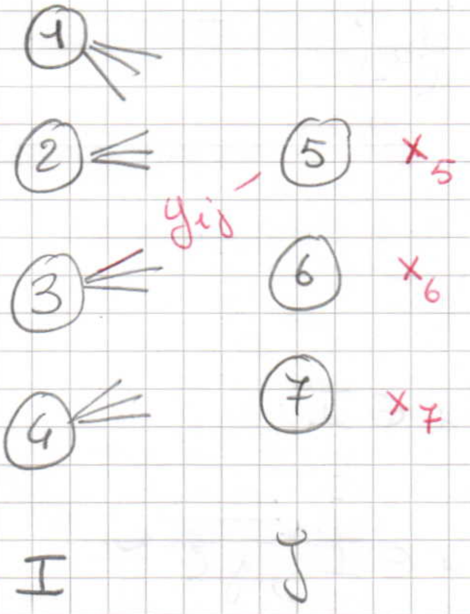
$$y_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J$$

Obs 1 : (location and allocation) variables as well as constraints are common to formulation (p-center) : only the objective function is different

Obs 2 : the formulation also minimizes the average travel distance (just divide the objective function by $|I|$)

Time complexity : NP-Hard for variable values of p

Example (p-median)



	5	6	7
1	20	25	60
2	30	15	10
3	30	15	10
4	30	15	10

d_{ij}

< complete graph >

let $p = 2$ and $h_i = 1$, $i = 1, 2, 3, 4$

$$\text{Min } 20\,y_{15} + 25\,y_{16} + 60\,y_{17} + 30\,y_{25} + 15\,y_{26} + 10\,y_{27} + 30\,y_{35} + 15\,y_{36} + 10\,y_{37} + 30\,y_{45} + 15\,y_{46} + 10\,y_{47}$$

Sub. to:

$$x_5 + x_6 + x_7 = 2$$

$$\left.\begin{array}{l} y_{15} + y_{16} + y_{17} = 1 \\ y_{25} + y_{26} + y_{27} = 1 \\ \vdots \end{array}\right\} \text{allocation constraints}$$

< the same for nodes 3 and 4 >

$$\left.\begin{aligned} y_{15} &\le x_5 \\ y_{16} &\le x_6 \\ y_{17} &\le x_7 \end{aligned}\right\}$$ linking between location and allocation

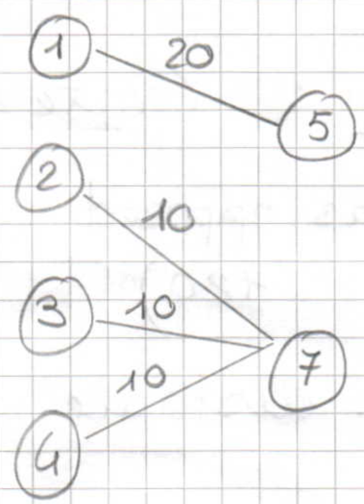
$\vdots$

< the same for the other links >

$$x_5, x_6, x_7 \in \{0, 1\}$$

$$y_{15}, y_{16}, y_{17} \dots \in \{0, 1\}$$

What is <u>an optimal solution</u> of the toy example? by inspection:



$$x^*_5 = x^*_7 = 1 \qquad x^*_6 = 0$$

$$y^*_{15} = y^*_{27} = y^*_{37} = y^*_{47} = 1$$

$$(y^*_{ij} = 0 \text{ other.})$$

with optimal value ≡ minimum total distance = 50

Observe that this solution is also optimal for the p-center problem defined on the same instance: the minimum maximum distance is 20!