

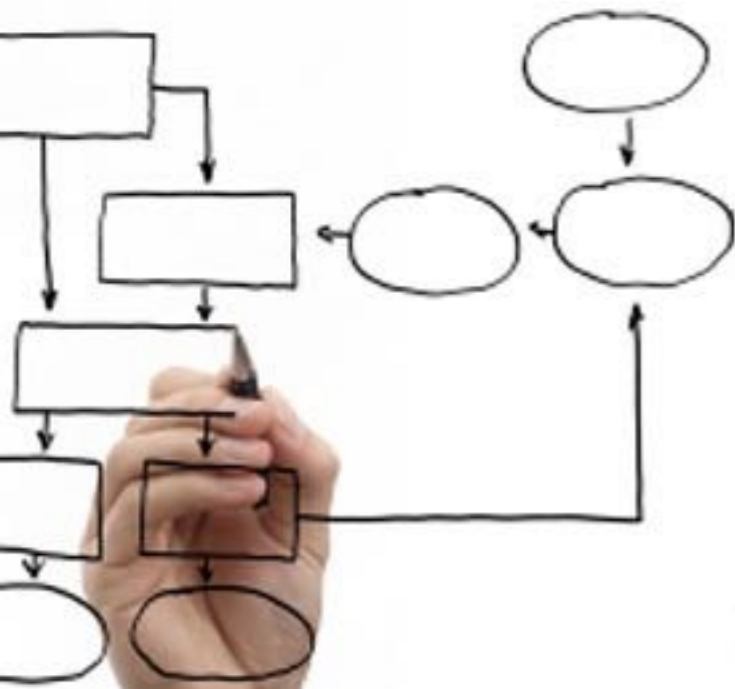
# Business Processes Modelling

## MPB (6 cfu, 295AA)

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12 - Strong Connectedness



# Object

$$N \vdash \psi$$

We survey  
two connectedness theorems and  
five exchange lemmas

Free Choice Nets (book, optional reading)

<https://www7.in.tum.de/~esparza/bookfc.html>

Two theorems on strong  
connectedness  
(whose proofs we omit)

# Strong connectedness theorem

**Theorem:** If a weakly connected system is live and bounded then it is strongly connected

# Consequences

If a (weakly-connected) net is not strongly connected

then

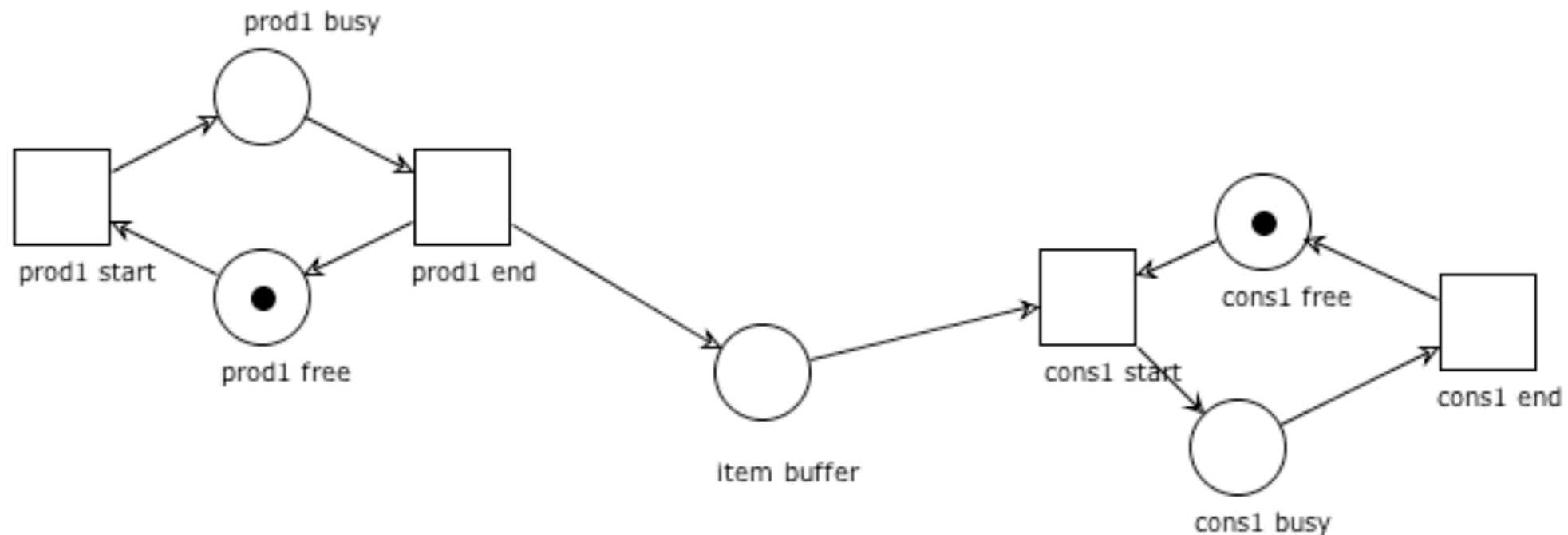
It is not “live and bounded”

If it is live, it is not bounded

If it is bounded, it is not live

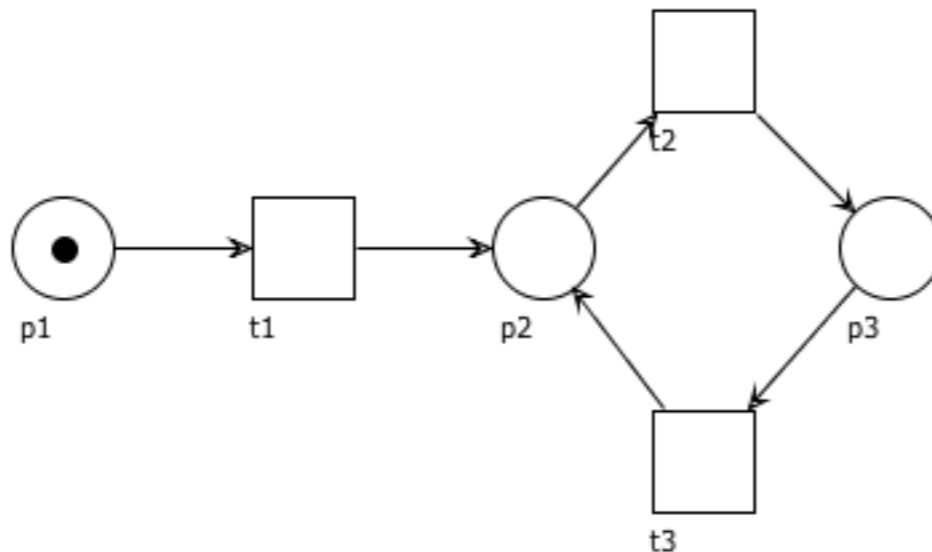
# Example

It is now immediate to see that this system  
(weakly connected, not strongly connected)  
cannot be live and bounded  
(it is live but not bounded)



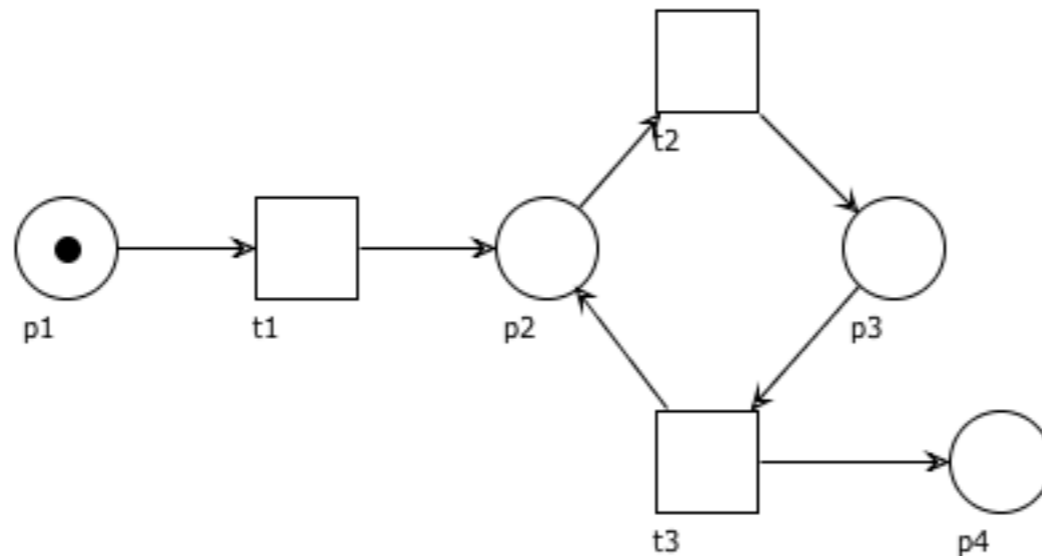
# Example

It is now immediate to see that this system  
(weakly connected, not strongly connected)  
cannot be live and bounded  
(it is bounded but not live)



# Example

It is now immediate to see that this system  
(weakly connected, not strongly connected)  
cannot be live and bounded  
(it is neither bounded nor live)





# Strong connectedness via invariants

**Theorem:** If a weakly connected net has a positive S-invariant  $I$  and a positive T-invariant  $J$  then it is strongly connected

# Consequences

If a (weakly-connected) net is not strongly connected

then

we cannot find (two) positive S- and T-invariants