

• current = \emptyset

In the nodes are indicated $\pi(u)$, whereas outside there is u .

min π	max π	$c(u,v)$	u	v
1	5	3	C	D
2	5	1	B	C
2	6	2	A	B
2	7	3	B	F
4	6	6	A	E
5	7	5	C	F
6	7	4	A	F

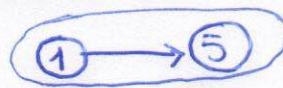
this is actually a priority queue



So we extract first by min π then by $c(u,v)$.

Extract min Q $\rightarrow \langle 1, 5, 3, C, D \rangle \rightarrow u=1, v=5$

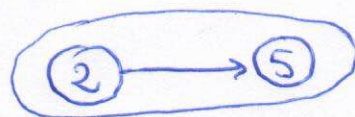
since current $\neq 1 = u \rightarrow$ current = 1, ~~current~~ $relinkTo = 5$
 output $\langle C, D \rangle$



then two nodes are merged as indicated

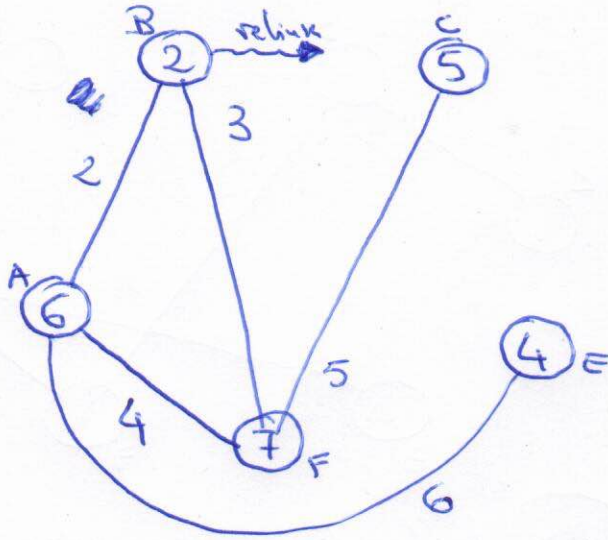
Extract min Q $\rightarrow \langle 2, 5, 1, B, C \rangle \rightarrow u=2, v=5$

since current $\neq 2 = u \rightarrow$ current = 2, $relinkTo = 5$
 output $\langle B, C \rangle$



all edges incident in 2 will be moved to 5 by the next steps.

We draw the graph for simplicity at this point:



min π	max π	c	u	v
2	6	2	A	B
2	7	3	B	F
4	6	6	A	E
5	7	5	C	F
6	7	4	A	F

current = 2, relinkTo = 5

The algorithm at this point proceeds as follows:

min Q \rightarrow $\langle 2, 6, 2, A, B \rangle \rightarrow u=2, v=6$

since current = 2 \Rightarrow we have to relink the edge changing $u \rightarrow$ relinkTo

Q.insert(5, 6, 2, A, B)

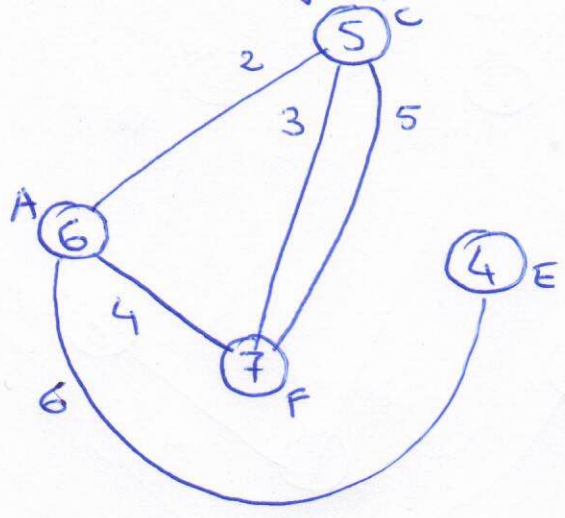
Before plotting the next graph and priority queue, let us do another step, since the next edge is also outgoing from 2 and thus it'll be relinkTo 5;

min Q \rightarrow $\langle 2, 7, 3, B, F \rangle \rightarrow u=2, v=7$

since current = 2 = u \Rightarrow we have to relink 2-7 to 5-7

Q.insert(5, 7, 3, B, F)

We draw the graph of this point, and the current PQ



min π	max π	c	u	v
4	6	6	A	E
5	6	2	A	B
5	7	3	B	F
5	7	5	C	F
6	7	4	A	F

current = 2, relinkTo = 5

the algorithm at this point does:

min Q \rightarrow $\langle 4, 6, 6, A, E \rangle \rightarrow u = 4, v = 6$

since current $\neq 4 = v \rightarrow$ we have to ~~relink~~ change



output $\langle A, E \rangle$

This way $4 \rightarrow 6$ hence every edge linked to 4 has to be relinked to 6, if any (we actually do not have them).

notice that ~~min Q~~ min Q is extracted and thus removed from Q

min Q \rightarrow $\langle 5, 6, 2, A, B \rangle \Rightarrow u = 5, v = 6$

since current $\neq 5 = u \rightarrow$ current = 5, relinkTo = 6

This way $5 \rightarrow 6$ hence every edge incident in 5 will be relinked to 6

output $\langle A, B \rangle$

extract min Q

min Q \rightarrow $\langle 5, 7, 3, B, F \rangle \rightarrow u=5, v=7$

since current = 5 = u \Rightarrow we have to rethink



Q. insert (6, 7, 3, B, F)

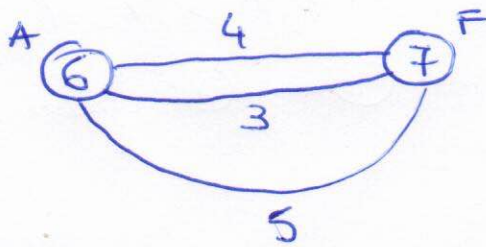
min Q \rightarrow $\langle 5, 7, 5, C, F \rangle \rightarrow u=5, v=7$

since current = 5 = u \Rightarrow we have to rethink



Q. insert (6, 7, 5, C, F)

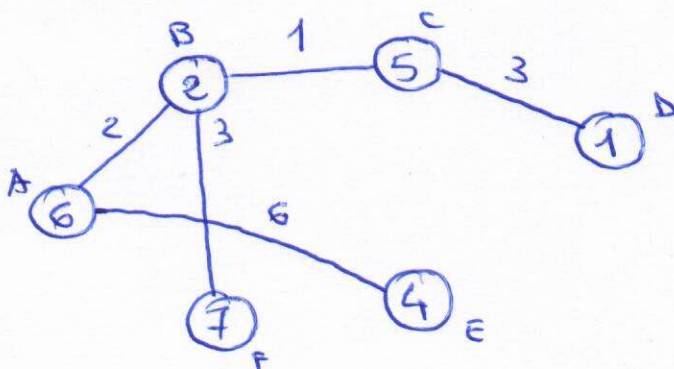
We draw the current graph and PQ



min	max	c	u	v
6	7	3	B	F
6	7	4	A	F
6	7	5	C	F

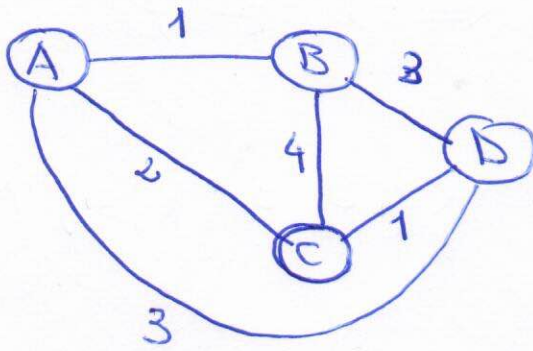
If $M=2$ then we can solve MST in memory of this graph hence output $\langle B, F \rangle$ which corresponds to

$\langle 6, 7, 3, B, F \rangle$
 \uparrow
 min weight



MST

Esercizio 2



current = ϕ
 π = Identical Perm.

min π	max π	cost	u	v
A	B	1	A	B
A	C	2	A	C
A	D	3	A	D
B	D	3	B	D
B	C	4	B	C
C	D	1	C	D

min Q $\rightarrow \langle A, B, 1, A, B \rangle \rightarrow u = A, v = B$

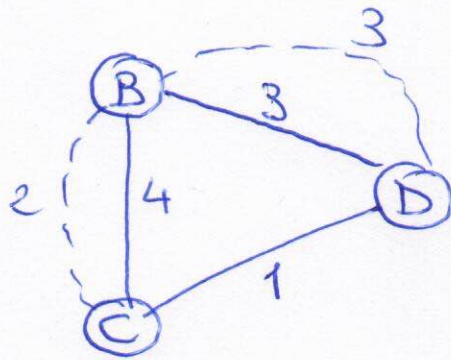
since current $\neq A = u \rightarrow$ current = A, relinkTo = B
 output $\langle A, B \rangle$
 extract min

min Q $\rightarrow \langle A, C, 2, A, C \rangle \rightarrow u = A, v = C$

since current = A = u \Rightarrow Q.insert(B, C, 2, A, C)
 [we have relinked $\textcircled{A} \rightarrow \textcircled{B}$]

$\min Q \rightarrow \langle A, D, 3, A, D \rangle \rightarrow u=A, v=D$
since current = A = u $\Rightarrow Q.inset(B, D, 3, A, D)$
 [we have relinked $A \rightarrow B$]
 =

The new graph is:



min π	max π	cost	u	v
B	C	2	A	C
B	C	4	B	C
B	D	3	B	D
B	D	3	A	D
C	D	1	C	D

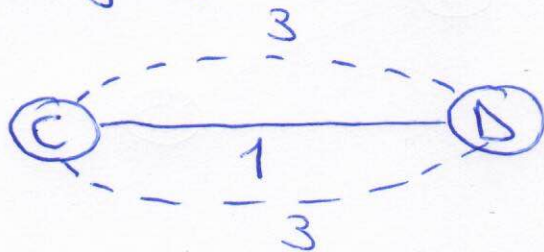
$\min Q \rightarrow \langle B, C, 2, A, C \rangle \Rightarrow u=B, v=C$
since current $\neq B = u \rightarrow$ current = B, relink to = C
 output $\langle A, C \rangle$
 extract min

$\min Q \rightarrow \langle B, C, 4, B, C \rangle \Rightarrow u=B, v=C$
since current = B = u \Rightarrow ~~current $\langle C, C, 4, B, C \rangle$~~
 no op because relink to = C = v.

$\min Q \rightarrow \langle B, D, 3, B, D \rangle \Rightarrow u = B, v = D$
since $\text{current} = B = u \Rightarrow Q.\text{insert}(C, D, 3, B, D)$
 $=$

$\min Q \rightarrow \langle B, D, 3, A, D \rangle \Rightarrow u = B, v = D$
since $\text{current} = B = u \Rightarrow Q.\text{insert}(C, D, 3, A, D)$
 $=$

The current graph and priority queue is:



min π	max π	cost	u	v
C	D	1	C	D
C	D	3	A	D
C	D	3	B	D

If $M=2$ then we apply Kruskal and thus
 output $\langle C, D, 1, C, D \rangle$

