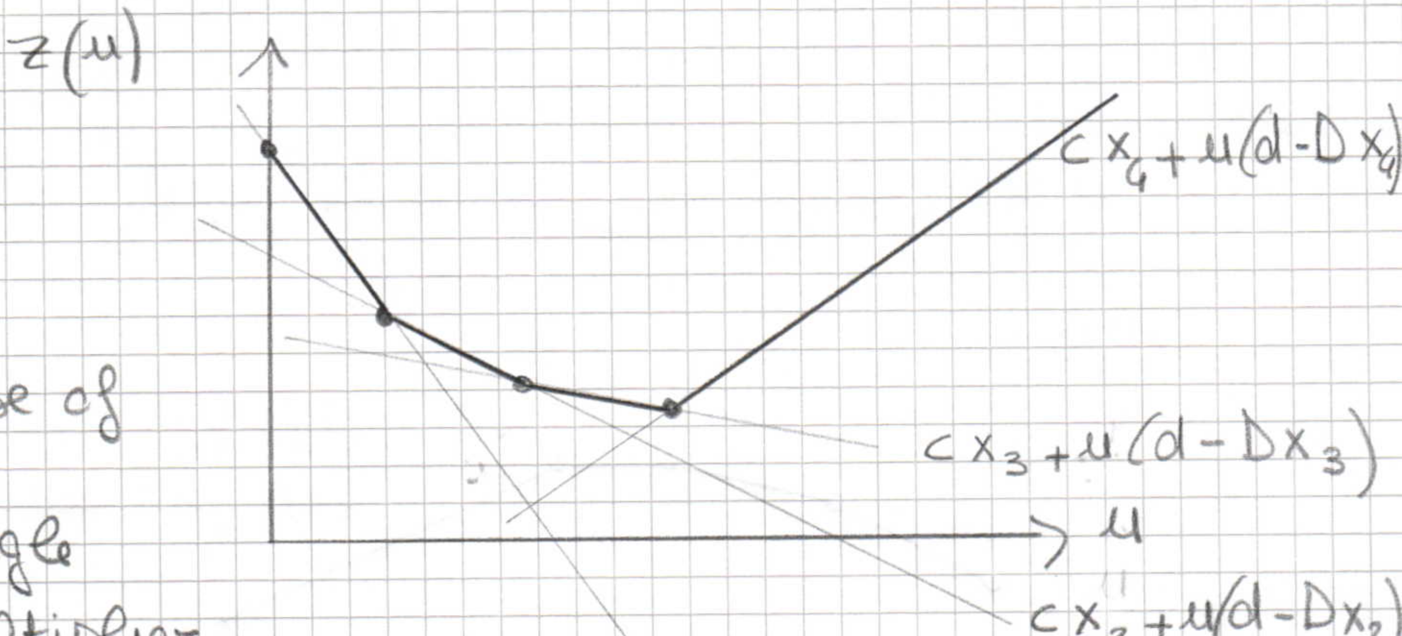
Solving the Lagrangian Dual

1

$$w_{LD} = \min_{u \geq 0} z(u)$$

$$= \min_{u \geq 0} \Big\{ \underbrace{\max_{t=1,..T} cx_t + u(d - Dx_t)}_{z(u)} \Big\}$$

i.e. the Lagrangian Dual consists in minimizing a piecewise linear convex, but nondifferentiable, function $z(u)$:



Case of a single multiplier u and $X = \{x_1, x_2, x_3, x_4\}$

How to minimize $z(u)$ over $u \geq 0$?

Solving the corresponding LP is not easy, since it has many (depending on T) constraints : <u>constraint generation approach</u>

<u>an example</u> : (CSP)

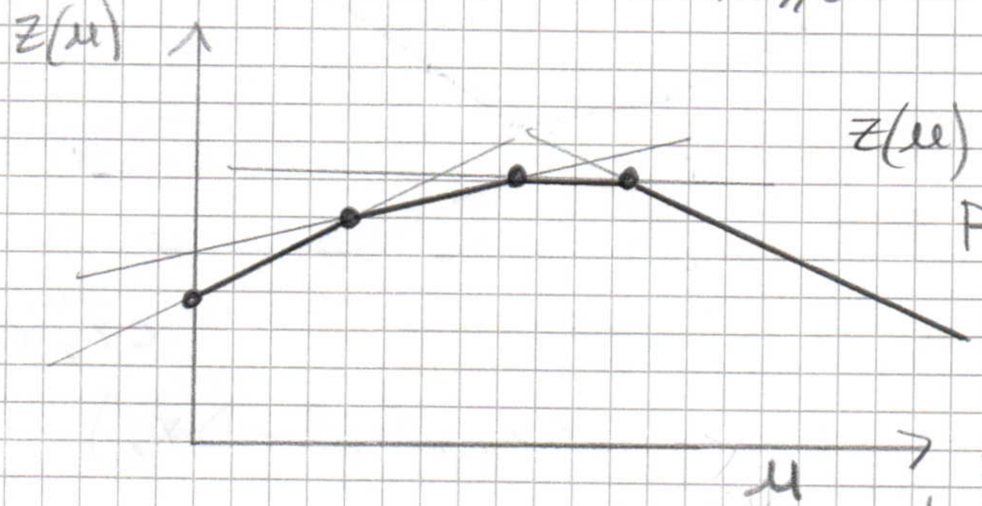
note that if (P) $z = \min cx$

$$Dx \leq d$$
$$x \in X$$

then (P_u) $z(u) = \min cx + u(Dx - d)$

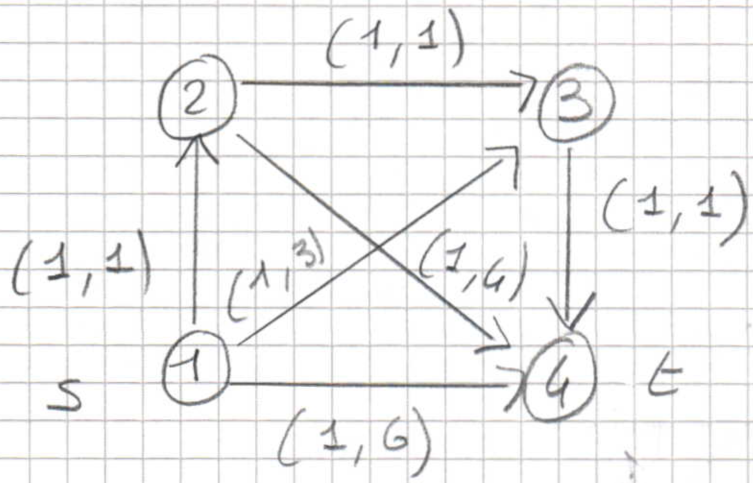
for $u \geq 0$ $\qquad x \in X$

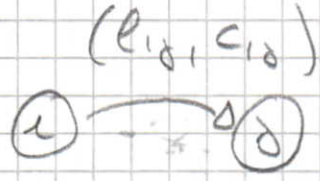
and (LD) $w_{LD} = \max_{u \geq 0} z(u)$



$z(u)$ is now a piecewise linear <u>concave</u> function, to be <u>maximized</u>

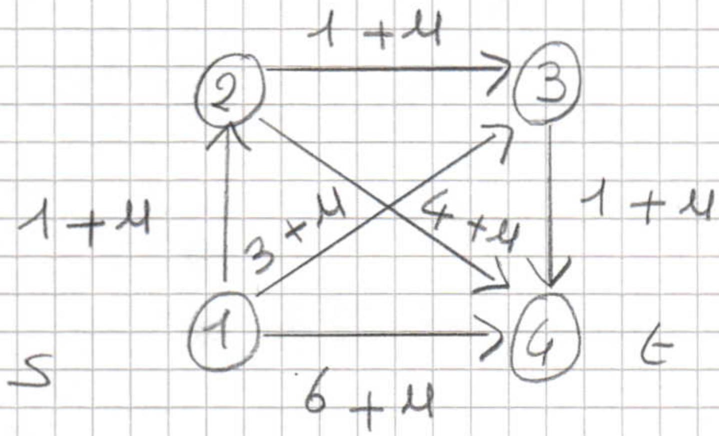
(CSP cont)





$L = 2$

consider the previous Lagrangian relaxation



modified costs

$c_{ij} + l_{ij} u$

- $z(u) = -Lu +$ "shortest path cost w.r.t. $\{c_{ij} + l_{ij} u\}$"
  $\quad = -2u$

- X is composed of 4 solutions : (T = 4 paths)

	modified cost	$\boxed{-2u}$
$P_1 = (1, 4)$	$6 + u$	$6 - u$
$P_2 = (1, 3, 4)$	$4 + 2u$	4
$P_3 = (1\ 2\ 4)$	$5 + 2u$	5
$P_4 = (1\ 2\ 3\ 4)$	$3 + 3u$	$3 + u$