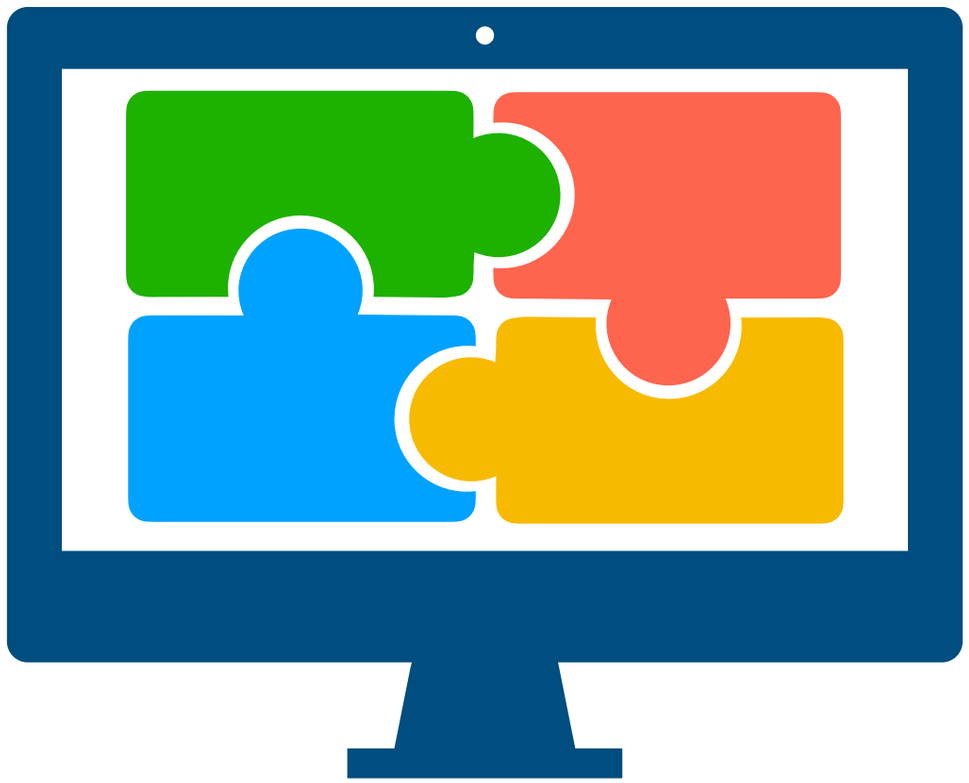


Linguaggi di Programmazione

Roberta Gori

http://didawiki.di.unipi.it/doku.php/magistraleinformatica/psc/

Applicazioni del CCS

CCS: sintassi

$$
\begin{array}{lcll}
p, q & ::= & \mathbf{nil} & \text{processo inattivo} \\
& | & x & \text{variabile di processo (per la ricorsione)} \\
& | & \mu.p & \text{prefisso azione} \\
& | & p\backslash\alpha & \text{canale ristretto} \\
& | & p[\phi] & \text{rietichettatura del canale} \\
& | & p + q & \text{scelta nondeterministica (somma)} \\
& | & p|q & \text{composizione parallela} \\
& | & \mathbf{rec}\ x.\ p & \text{ricorsione}
\end{array}
$$

(gli operatori sono elencati in ordine di precedenza)

Un po' di notazione

scriviamo $\sum_{i=1}^{n} p_i$ invece di $p_1 + \cdots + p_n$

scriviamo $\prod_{i=1}^{n} p_i$ invece di $p_1 | \cdots | p_n$

scriviamo $p \backslash \{a_1, ..., a_n\}$ invece di $p \backslash a_1 \cdots \backslash a_n$

scriviamo $\mu^n.p$ invece di $\underbrace{\mu.\mu....\mu.}_{n} p$

CCS op. semantica

Act) $$\frac{}{\mu.p \xrightarrow{\mu} p}$$

Res) $$\frac{p \xrightarrow{\mu} q \quad \mu \notin \{\alpha, \overline{\alpha}\}}{p\backslash\alpha \xrightarrow{\mu} q\backslash\alpha}$$

Rel) $$\frac{p \xrightarrow{\mu} q}{p[\phi] \xrightarrow{\phi(\mu)} q[\phi]}$$

SumL) $$\frac{p_1 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

SumR) $$\frac{p_2 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

ParL) $$\frac{p_1 \xrightarrow{\mu} q_1}{p_1|p_2 \xrightarrow{\mu} q_1|p_2}$$

Com) $$\frac{p_1 \xrightarrow{\lambda} q_1 \quad p_2 \xrightarrow{\overline{\lambda}} q_2}{p_1|p_2 \xrightarrow{\tau} q_1|q_2}$$

ParR) $$\frac{p_2 \xrightarrow{\mu} q_2}{p_1|p_2 \xrightarrow{\mu} p_1|q_2}$$

Rec) $$\frac{p\left[{}^{\mathbf{rec}\ x.\ p}/_{x}\right] \xrightarrow{\mu} q}{\mathbf{rec}\ x.\ p \xrightarrow{\mu} q}$$

CCS

Codificare un linguaggio imperativo

Prima cosa: terminazione

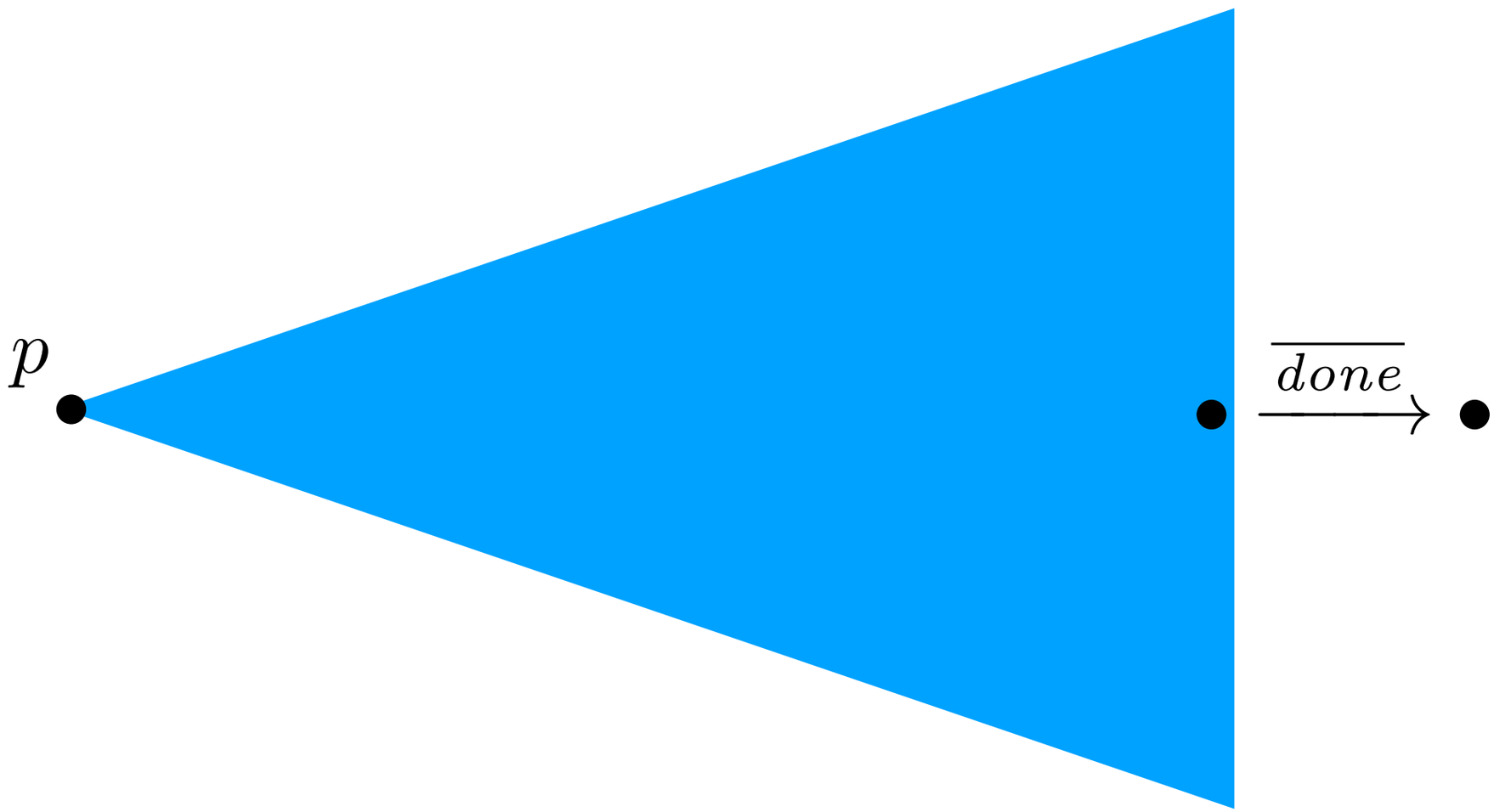
Un canale dedicato *done*:

viene inviato un messaggio quando il comando corrente termina

$$\text{Done} \triangleq \overline{done}$$

$$\text{Done} \xrightarrow{\overline{done}} \mathbf{nil}$$

Terminazione



Skip

skip

non fa nulla invia *done*

$$\tau.\text{Done}$$

$$\bullet \xrightarrow{\tau} \text{Done} \xrightarrow{\overline{done}} \mathbf{nil}$$

Variabili

x che varia su $V = \{v_1, ..., v_n\}$

un processo dedicato alla gestione di ogni variabile

possiamo leggere il suo valore attuale (xr_i canale)

possiamo scrivere qualsiasi valore (canale xw_i)

$$\mathsf{XW} \triangleq \sum_{i=1}^{n} xw_i.\mathsf{X}_i$$
$$= xw_1.\mathsf{X}_1 + \cdots + xw_n.\mathsf{X}_n$$

$$\mathsf{X}_i \triangleq \overline{xr}_i.\mathsf{X}_i + \mathsf{XW}$$

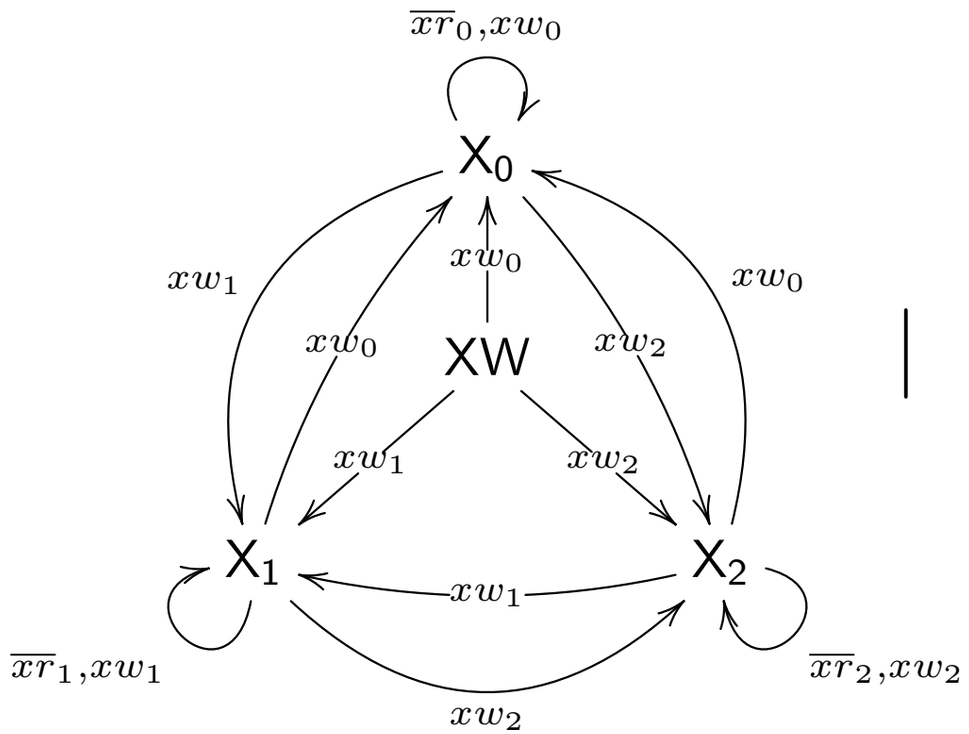
Dichiarazione di variabili

var x

rilascia una variabile non inizializzata e termina

XW|Done

$V = \{v_0, v_1, v_2\}$



|

Done $\xrightarrow{\overline{done}}$ **nil**

Assignamento

$$x := v_i$$

invia un messaggio per cambiare lo stato della variabile
e poi termina

$$\overline{xw}_i.\mathsf{Done}$$

$$\bullet \xrightarrow{\overline{xw}_i} \mathsf{Done} \xrightarrow{\overline{done}} \mathbf{nil}$$

Composizione sequenziale

$$c_1 \; ; \; c_2$$

assumiamo p_1 modella c_1

p_2 modella c_2

$$p_1 | done.p_2$$

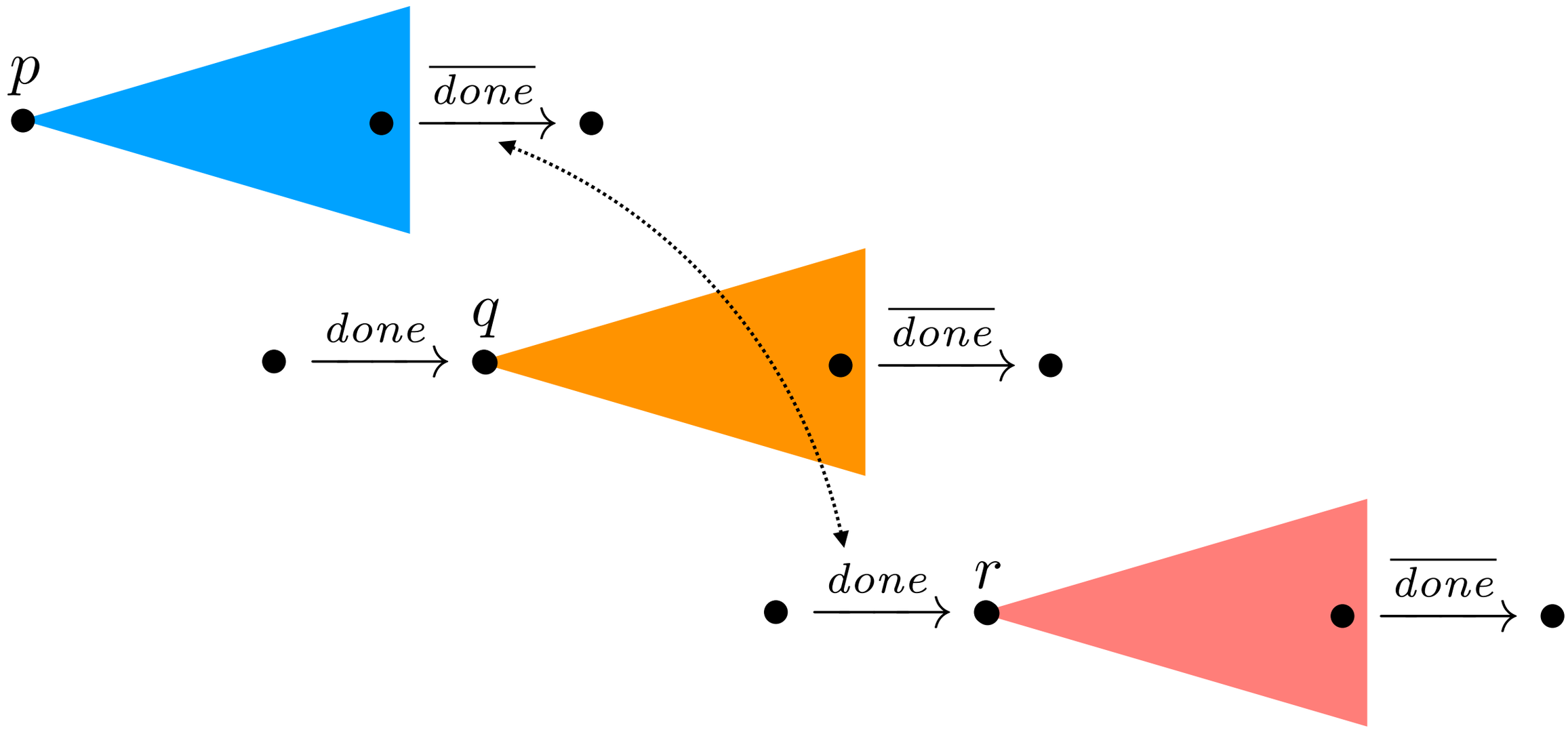
scelta infelice: non è scalabile

$$c_1; c_2; c_3$$

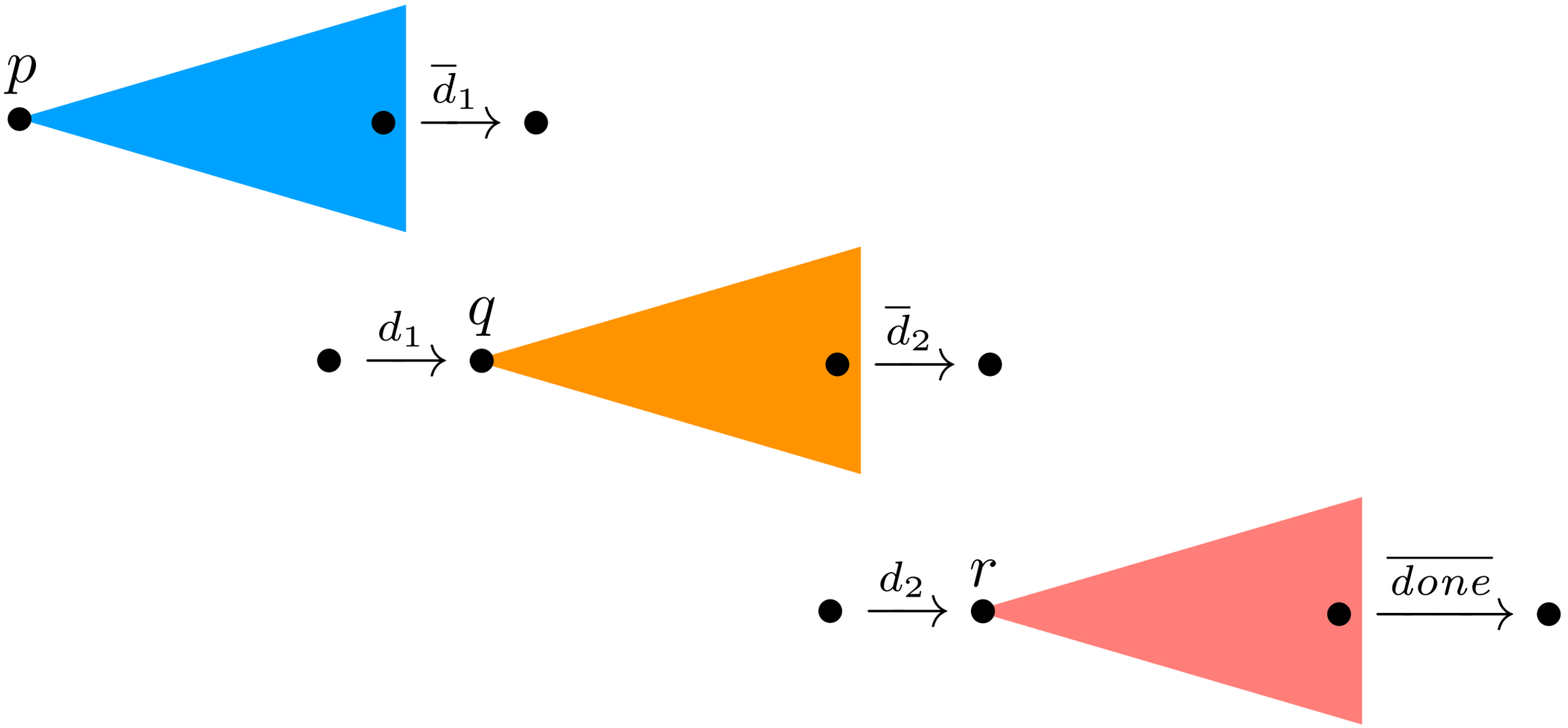
$$p_1 | done.p_2 | done.p_3$$

p_3 puo' iniziare dopo p_1

Sequenziale?



Sequential!



Composizione sequenziale

$$c_1 \; ; \; c_2$$

assumiamo p_1 modella c_1

p_2 modella c_2

$$\phi_d(done) = d$$

$$p_1 \frown p_2 \triangleq (p_1[\phi_d] | d.p_2) \backslash d$$

ora d e' locale a p_1 e p_2

$$(\; ((p_1[\phi_{d_1}] \mid d_1.p_2) \backslash d_1)[\phi_{d_2}] \mid d_2.p_3) \backslash d_2$$

$$(\; ((p_1[\phi_d] \mid d.p_2) \backslash d)[\phi_d] \mid d.p_3) \backslash d$$

$$(p_1 \frown p_2) \frown p_3$$

Condizionale

if $x = v_i$ then c_1 else c_2

assumiamo p_1 modella c_1

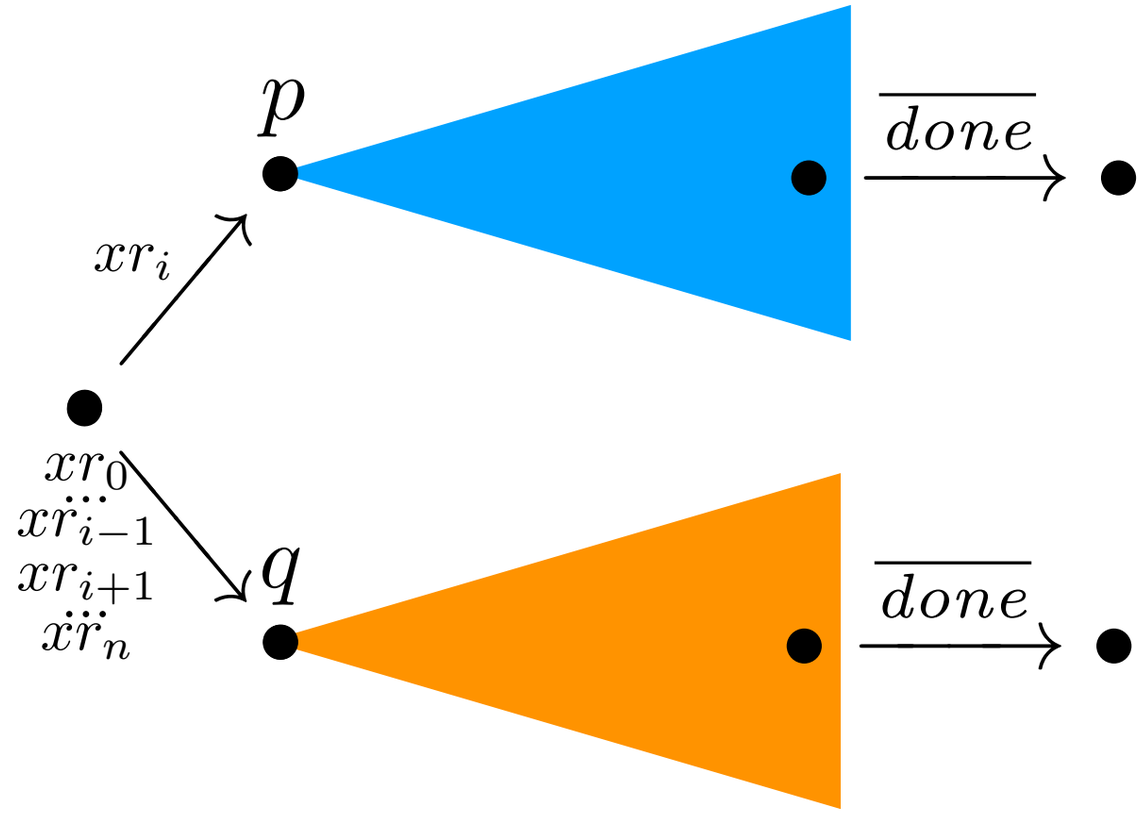
p_2 modella c_2

riceve lo stato della variabile

e poi sceglie di conseguenza

$$xr_i.p_1 + \sum_{j \neq i} xr_j.p_2$$

Condizionale



Iterazione

while $x = v_i$ do c

assumiamo p modella c

riceve lo stato della variabile
e poi sceglie di conseguenza, eventualmente ricorrendo

$$\mathbf{rec}\ y.\ xr_i.(p[\phi_d]|d.y)\backslash d + \sum_{j \neq i} xr_j.\mathsf{Done}$$

$$Y \triangleq xr_i.(p[\phi_d]|d.Y)\backslash d + \sum_{j \neq i} xr_j.\mathsf{Done}$$

$$Y \triangleq xr_i.(p \frown Y) + \sum_{j \neq i} xr_j.\mathsf{Done}$$

Riassumendo

tutti i canali di comunicazione con le variabili
devono essere ristretti
per garantire che le richieste di lettura/scrittura siano sincronizzate

$$p\backslash\{xw_1, xr_1, \cdots\}$$

sono possibili diverse ottimizzazioni:
prefisso d'azione invece del collegamento per la composizione sequenziale
guardie più espressive
rimuovere le transizioni silenziose
introdurre costanti per i cicli
implementare espressioni

Example: optimisation

$$x := 1; y := 2$$

$$\overline{xw}_1.\overline{done} \quad \frown \quad \overline{yw}_2.\overline{done}$$

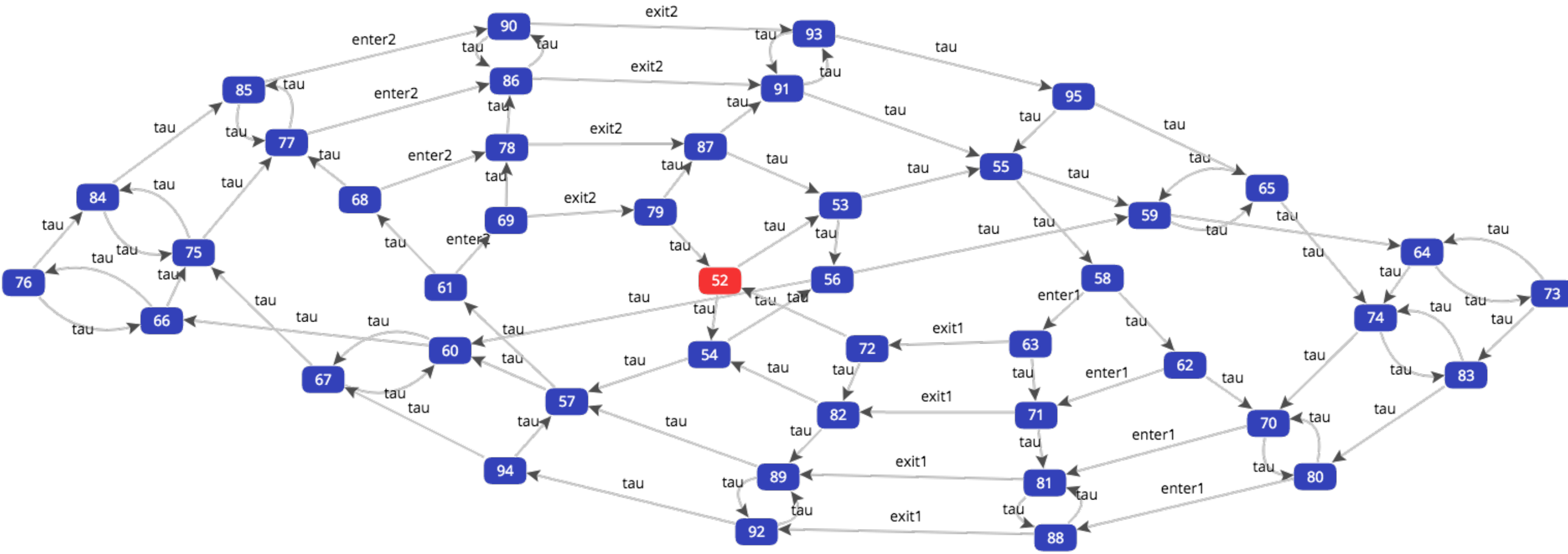
$$(\ (\overline{xw}_1.\overline{done})[\phi_d] \mid d.\overline{yw}_2.\overline{done}\ )\backslash d$$

$$\overline{xw}_1.\overline{yw}_2.\overline{done}$$

CCS

Giochiamo con CAAL

Concurrency Workbench, Aalborg Edition



http://caal.cs.aau.dk//

CAAL

CAAL è uno strumento basato sul web per
la modellazione, la visualizzazione e la verifica
di processi concorrenti in CCS (e la sua estensione temporale)

sviluppato per scopi educativi

Edit permette di specificare processi (definiti ricorsivamente)

Explore i loro LTS (collassando stati equivalenti)

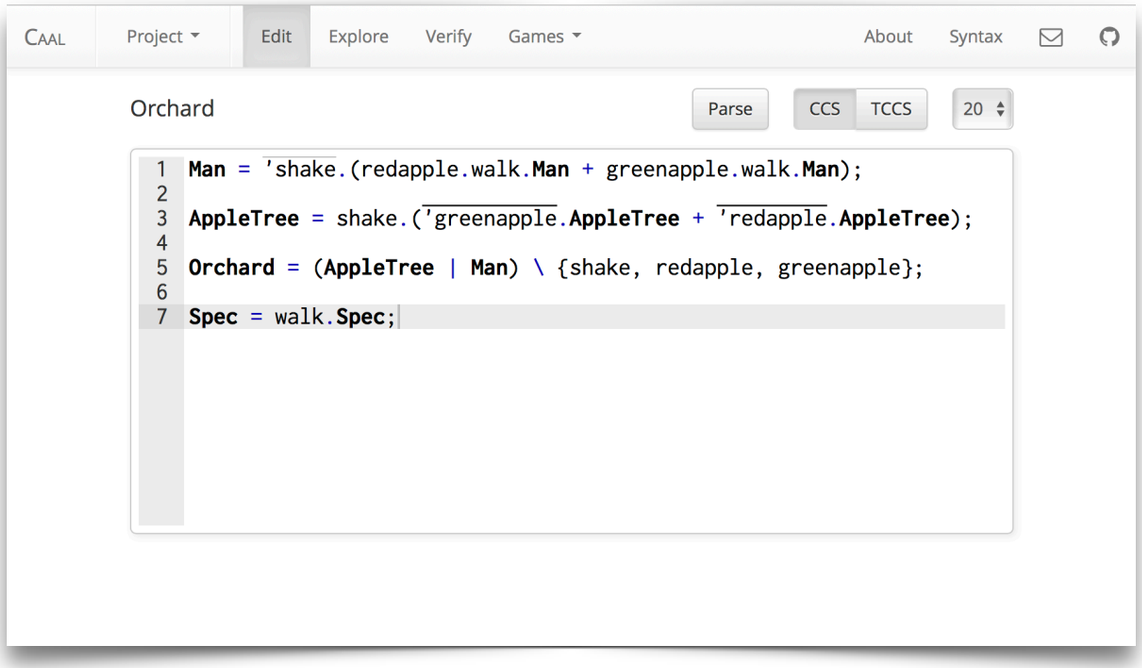
Verify controlla la soddisfazione delle formule HML (model checking)

Verify equivalenza tra processi

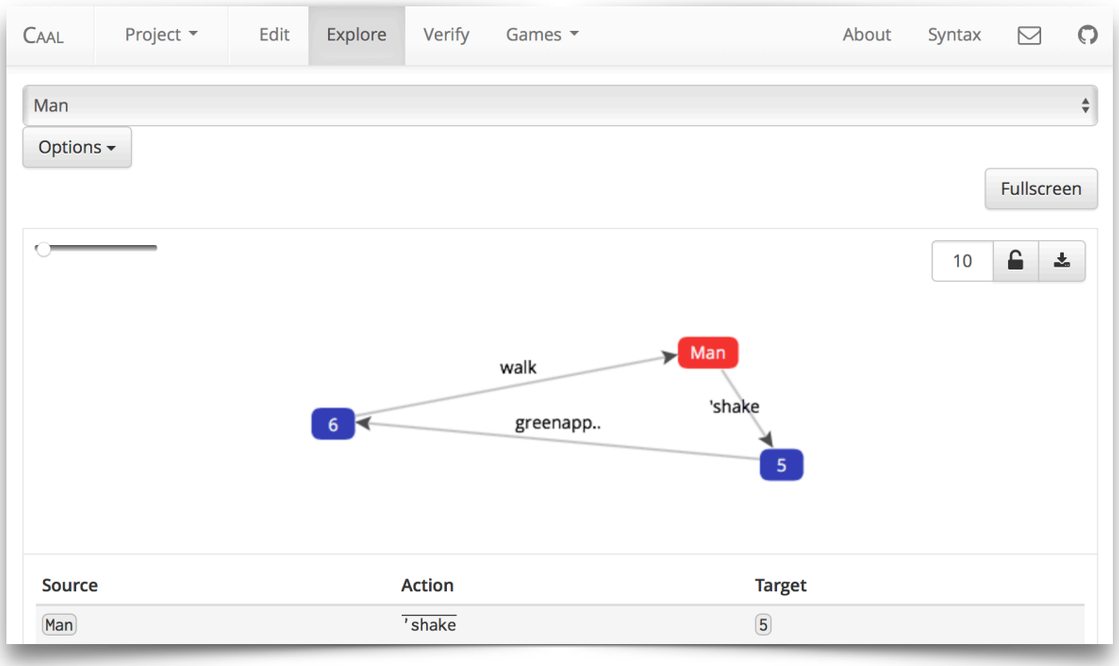
permette di giocare al **Game** della simulazione

CAAL

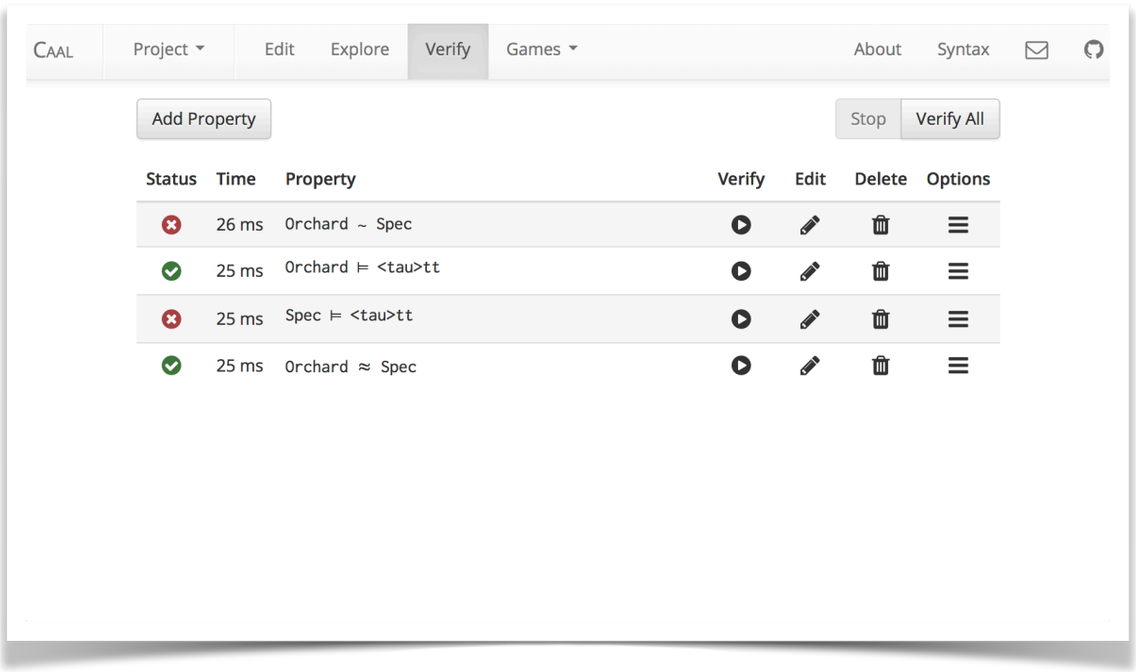
Edit processes



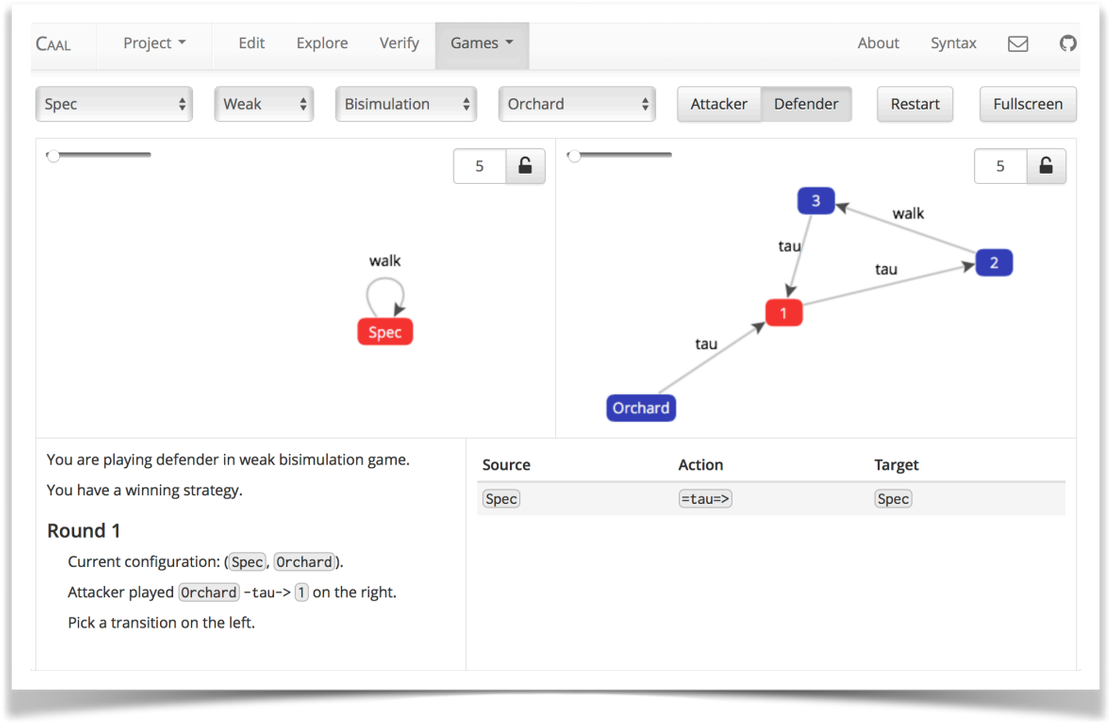
Explore LTS



Verify HML & Equivalences



play bisimulation **Game**



CAAL sintassi per CCS

$\mathbf{nil}$	0
a.P $\quad \overline{a}.P \quad \tau.P$	a.P ‘a.P tau.P
$P + Q$	P + Q
$P \mid Q$	P | Q
$P \backslash \{a1, \ldots, an\}$	P \{a1,…,an}
$P[^{b1}/_{a1}, \ldots, ^{bn}/_{an}]$	P[b1/a1,…,bn/an]
$A \triangleq P$	A = P ;

CAAL sintassi per HML

$\mathbf{tt}$	tt
$\mathbf{ff}$	ff
$F \wedge G$	F and G
$F \vee G$	F or G
$\Diamond_{\{a1,\ldots,an\}} F$	$\langle$a1,...,an$\rangle$F
$\Box_{\{a1,\ldots,an\}} F$	[a1,...,an]F

CAAL

mutual exclusion protocols analysis

Peterson's mex algorithm

```
% Two processes P1, P2
% Two boolean variables b1, b2 (both initially false)
% when Pi wants to enter the critical section, then it sets bi to true
% An integer variable k, taking values in {1,2}
% (initial value is arbitrary)
% the process Pk has priority over the other process
%
% Process P1 in pseudocode
```

```
while (true) {
    ...                          % non critical section
    b1 = true ;                  % P1 wants to enter the critical section
    k = 2 ;                      % P1 gives priority to the other process
    while (b2 && k==2) skip ;    % P1 waits its turn
    ...                          % P1 enters the critical section
    b1 = false                   % P1 leaves the critical section
}
```

```
% Process P2 is analogous to P1
```

Peterson's mex in CCS

$$\mathsf{B1W} \triangleq b1wf.\mathsf{B1f} + b1wt.\mathsf{B1t}$$
$$\mathsf{B1f} \triangleq \overline{b1rf}.\mathsf{B1f} + \mathsf{B1W}$$
$$\mathsf{B1t} \triangleq \overline{b1rt}.\mathsf{B1t} + \mathsf{B1W}$$

$$\mathsf{KW} \triangleq kw1.\mathsf{K1} + kw2.\mathsf{K2}$$
$$\mathsf{K1} \triangleq \overline{kr1}.\mathsf{K1} + \mathsf{KW}$$
$$\mathsf{K2} \triangleq \overline{kr2}.\mathsf{K2} + \mathsf{KW}$$

```
% Process P1 in pseudocode
while (true) {
    ...
    b1 = true ; % wants to enter
    k = 2 ; % gives priority to P2
    while (b2 && k==2) skip ; % waits
    ...  % enters critical section
    b1 = false % leaves
}

% Process P2 is analogous to P1
```

$$\mathsf{P1} \triangleq \overline{b1wt}.\overline{kw2}.\mathsf{P1a}$$
$$\mathsf{P1a} \triangleq b2rf.\mathsf{P1b} + b2rt.(kr1.\mathsf{P1b} + kr2.\mathsf{P1a})$$
$$\mathsf{P1b} \triangleq enter1.exit1.\overline{b1wf}.\mathsf{P1}$$

leave cycle

loop

just something to observe

$$\mathsf{SP} \triangleq (\mathsf{B1f}|\mathsf{B2f}|\mathsf{KW}|\mathsf{P1}|\mathsf{P2})\backslash\{kw1, kr1, kw2, kr2, ...\}$$

Peterson's mex in CCS

$$\mathsf{B1W} \triangleq b1wf.\mathsf{B1f} + b1wt.\mathsf{B1t}$$

$$\mathsf{B1f} \triangleq \overline{b1rf}.\mathsf{B1f} + \mathsf{B1W}$$

$$\mathsf{B1t} \triangleq \overline{b1rt}.\mathsf{B1t} + \mathsf{B1W}$$

$$\mathsf{KW} \triangleq kw1.\mathsf{K1} + kw2.\mathsf{K2}$$

$$\mathsf{K1} \triangleq \overline{kr1}.\mathsf{K1} + \mathsf{KW}$$

$$\mathsf{K2} \triangleq \overline{kr2}.\mathsf{K2} + \mathsf{KW}$$

```
% Process P1 in pseudocode
while (true) {
     ...
     b1 = true ; % wants to enter
     k = 2 ; % gives priority to P2
     while (b2 && k==2) skip ; % waits
     ...  % enters critical section
     b1 = false % leaves
}

% Process P2 is analogous to P1
```

$$\mathsf{P1} \triangleq \overline{b1wt}.\overline{kw2}.\mathsf{P1a}$$

no busy wait

$$\mathsf{P1a} \triangleq b2rf.\mathsf{P1b} + kr1.\mathsf{P1b}$$

$$\mathsf{P1b} \triangleq enter1.exit1.\overline{b1wf}.\mathsf{P1}$$

$$\mathsf{SP} \triangleq (\mathsf{B1f}|\mathsf{B2f}|\mathsf{KW}|\mathsf{P1}|\mathsf{P2})\backslash\{kw1, kr1, kw2, kr2, ...\}$$

Peterson's mex in CCS

P1 $\triangleq \overline{b1wt}.\overline{kw2}$.P1a

P1a $\triangleq b2rf$.P1b $+ kr1$.P1b

P1b $\triangleq enter1.exit1.\overline{b1wf}$.P1

SP $\triangleq$ (B1f|B2f|KW|P1|P2)$\backslash\{kw1, kr1, kw2, kr2, ...\}$

una formula per la mutua esclusione?

qualsiasi label

$F \triangleq [exit_1]\mathbf{ff} \vee [exit_2]\mathbf{ff}$

$MEX \underset{\text{(max)}}{\triangleq} F \wedge [-]MEX$

$F \wedge [-](\cdots)$

$F \wedge [-](F \wedge [-](\cdots))$

$F \wedge [-](F \wedge [-](F \wedge [-](\cdots)))$

una formula definita ricorsivamente!

Peterson's mex in CCS

$\mathsf{P1} \triangleq \overline{b1wt}.\overline{kw2}.\mathsf{P1a}$

$\mathsf{P1a} \triangleq b2rf.\mathsf{P1b} + kr1.\mathsf{P1b}$

$\mathsf{P1b} \triangleq enter1.exit1.\overline{b1wf}.\mathsf{P1}$

$\mathsf{SP} \triangleq (\mathsf{B1f}|\mathsf{B2f}|\mathsf{KW}|\mathsf{P1}|\mathsf{P2})\backslash\{kw1, kr1, kw2, kr2, ...\}$

P1 ha la possibilita' di entrare?

$G \triangleq \langle enter_1 \rangle \mathbf{tt}$

$EN \underset{\text{(min)}}{\triangleq} G \vee \langle - \rangle EN$

$G \vee \langle - \rangle(\cdots)$

$G \vee \langle - \rangle(G \vee \langle - \rangle(\cdots))$

$G \vee \langle - \rangle(G \vee \langle - \rangle(G \vee \langle - \rangle(\cdots)))$

una formula definita ricorsivamente!

Peterson's mex in CCS

P1 $\triangleq \overline{b1wt}.\overline{kw2}$.P1a

P1a $\triangleq b2rf$.P1b $+ kr1$.P1b

P1b $\triangleq enter1.exit1.\overline{b1wf}$.P1

SP $\triangleq$ (B1f|B2f|KW|P1|P2)$\backslash\{kw1, kr1, kw2, kr2, ...\}$

non abbiamo deadlock?

$H \triangleq \langle - \rangle \mathbf{tt}$

$DF \underset{\text{(max)}}{\triangleq} H \wedge [-]DF$

$H \wedge [-](\cdots)$

$H \wedge [-](H \wedge [-](\cdots))$

$H \wedge [-](H \wedge [-](H \wedge [-](\cdots)))$

una formula definita ricorsivamente!

Peterson's mex in CCS

P1 $\triangleq \overline{b1wt}.req_1.\overline{kw2}$.P1a

P1a $\triangleq b2rf$.P1b $+ kr1$.P1b

P1b $\triangleq enter1.exit1.\overline{b1wf}$.P1

SP $\triangleq$ (B1f|B2f|KW|P1|P2)$\backslash\{kw1, kr1, kw2, kr2, ...\}$

P1 puo' entrare ogni volta che fa una richiesta?

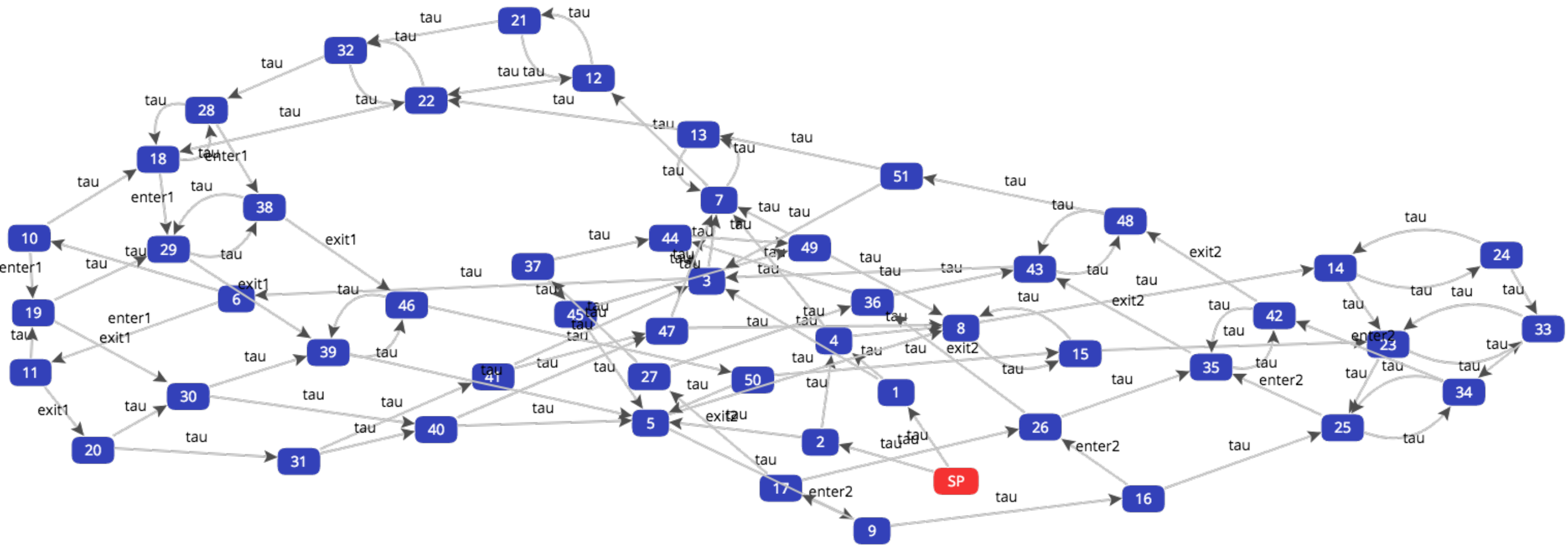
$$A \underset{\text{(min)}}{\triangleq} \langle exit_1 \rangle \mathbf{tt} \vee [-]A$$

$$REQ \underset{\text{(max)}}{\triangleq} [req_1]A \wedge [-]REQ$$

una formula definita ricorsivamente!

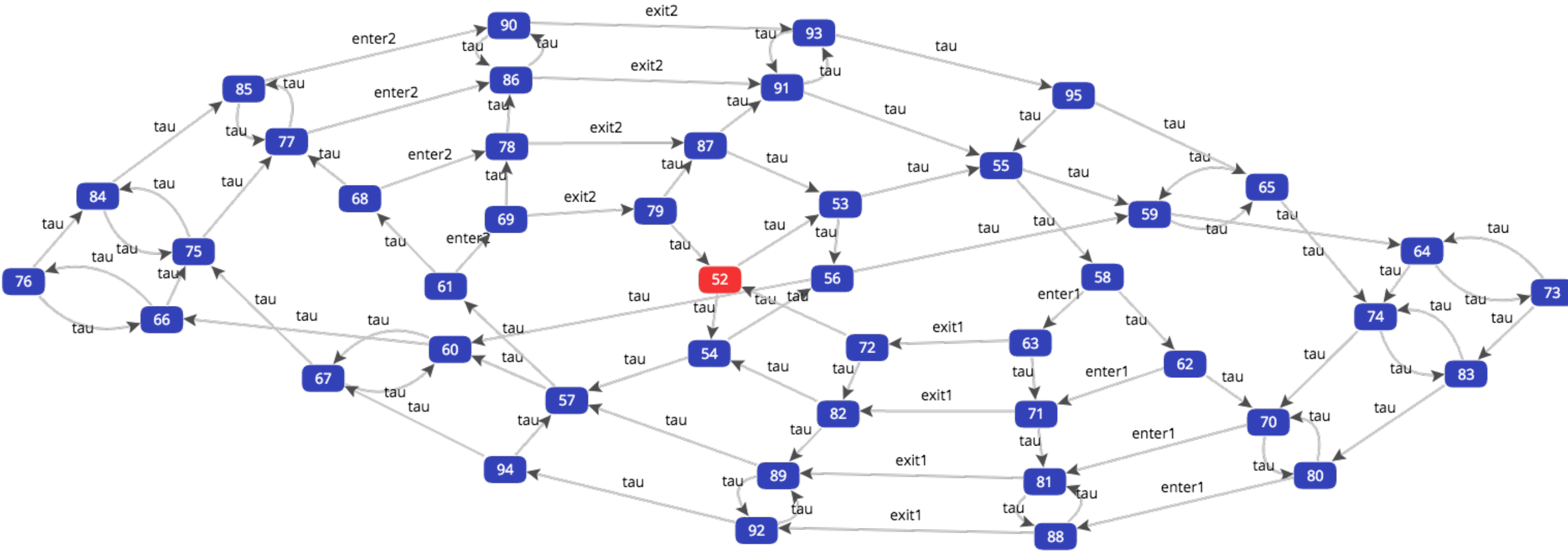
Peterson's mex in CAAL

LTS



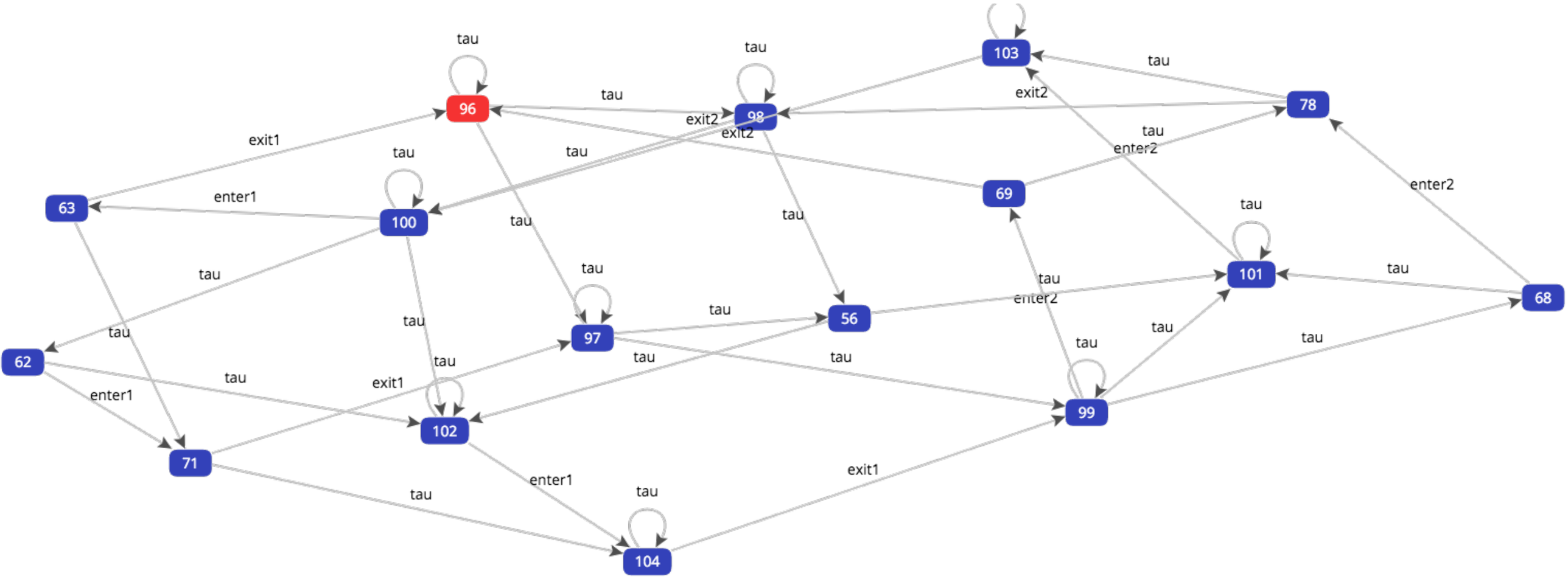
Peterson's mex in CAAL

LTS up to strong bisimilarity



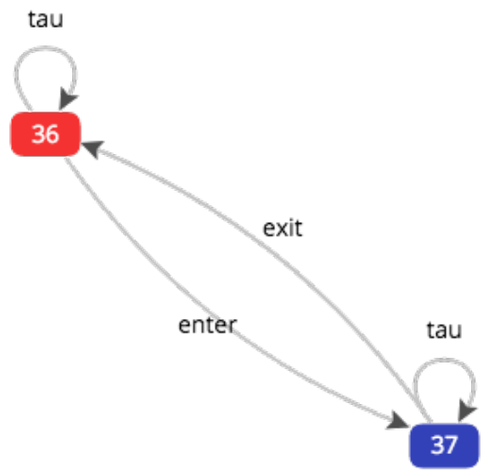
Peterson's mex in CAAL

LTS up to weak bisimilarity



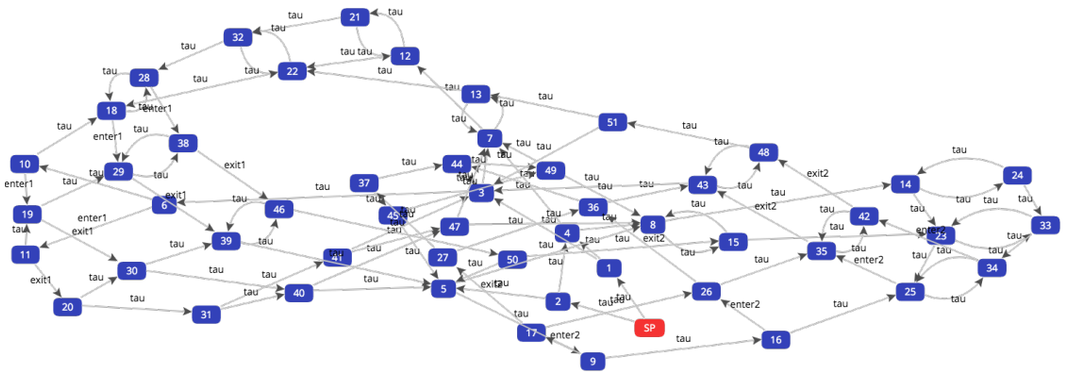
Peterson's mex in CAAL

LTS up to weak bisimilarity, after renaming *enter1/2* to *enter* and *exit1/2* to *exit*

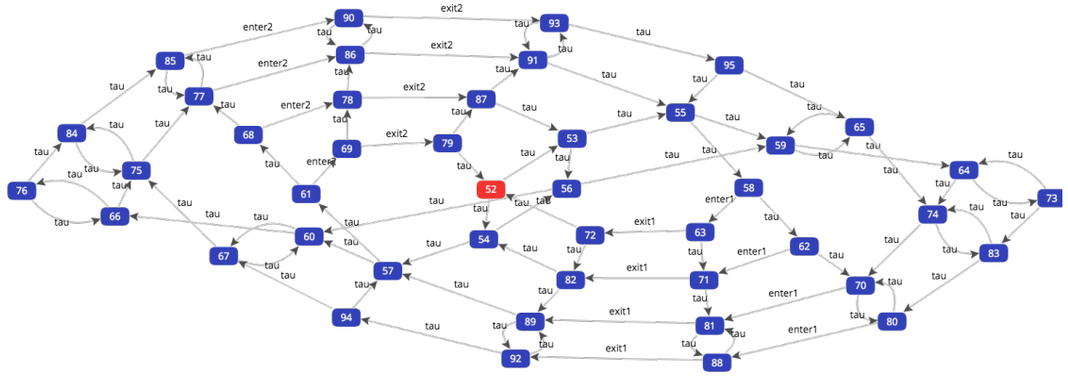


Peterson's mex in CAAL

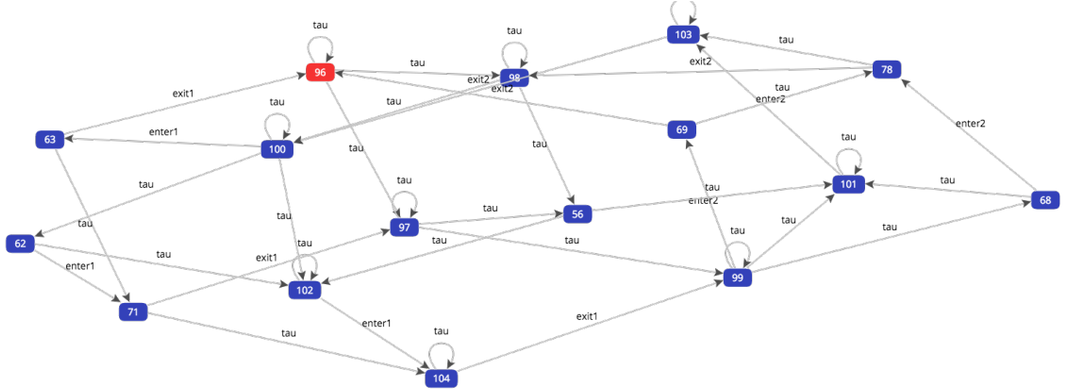
LTS



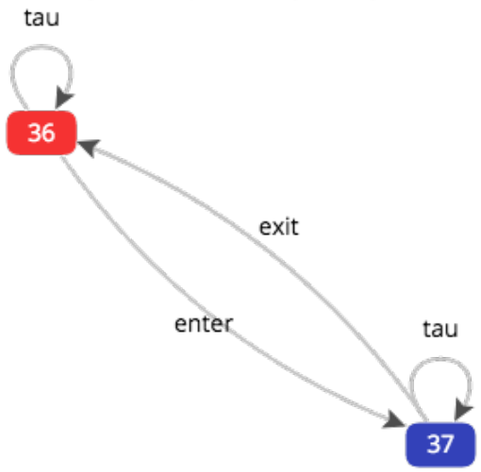
LTS up to strong bisimilarity



LTS up to weak bisimilarity



LTS up to weak bisimilarity, *enter* and *exit*



Hyman's mex algorithm

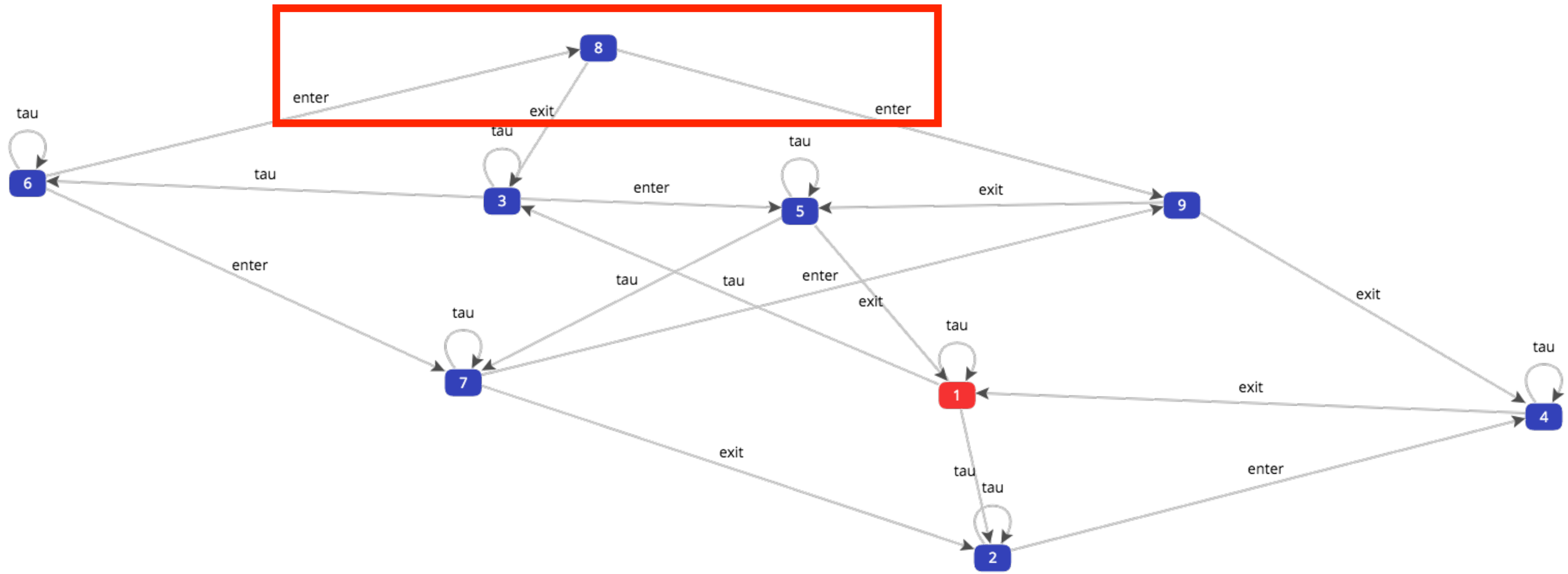
```
% Two processes H1, H2
% Two boolean variables b1, b2 (both initially false)
% when Hi wants to enter the critical section, then it sets bi to true
% An integer variable k, taking values in {1,2}
% (initial value is arbitrary)
% the process Hk has priority over the other process
%
% Process H1 in pseudocode
```

```
while (true) {
    ...                          % non critical section
    b1 = true ;                  % H1 wants to enter the critical section
    while (k==2) {               % while H2 has priority
        while (b2) skip ;        % H1 waits
        k = 1;                   % H1 sets priority to itself
    }
    ...                          % H1 enters the critical section
    b1 = false                   % H1 leaves the critical section
}
```

```
% Process H2 is analogous to H1
```

Peterson's mex in CAAL

LTS up to weak bisimilarity, after renaming *enter1/2* to *enter* and *exit1/2* to *exit*



50 prisoners puzzle

50 prisoners kept in separate cells got a chance to be released:
from time to time one of them will be carried in a special room and then back to cell.
(in no particular order, possibly multiple times consecutively, but fairly to avoid infinite wait)

The room is completely empty except for a switch that can turn the light on or off
(the light is not visible from outside).

At any time, if any of them claims rightfully that all the prisoners have already entered the room at least once, then all prisoners will be released
(but if it proves wrong, then the chance ends and they will never be released).

The prisoners have the possibility to discuss in advance some protocol to follow
(not all prisoner must behave in the same way).

Can you find a winning strategy for the prisoners?
Can you formalise it in CCS (for 2 to 4 prisoners)?

- Easy case: it is known that the light in the room is initially off
- Hard case: the initial state of the light in the room is not known.