(1) [redacted]

reset x in c

(1.1)
$$\frac{\langle c, \sigma[0/x]\rangle \to \sigma'}{\langle \text{reset } x \text{ in } c, \sigma\rangle \longrightarrow \sigma'[\sigma x/x]}$$

(1.2) $\mathcal{C}[\![\text{reset } x \text{ in } c]\!]\sigma = \left(\mathcal{C}[\![c]\!](\sigma[0/x])\right)[\sigma x/x]$ where $[{}^{v}/_{x}]$ is extended to deal with $\perp$

(1.3) a) $P(\langle c, \sigma\rangle \to \sigma') \overset{def}{=} \mathcal{C}[\![c]\!]\sigma = \sigma'$ by rule induction

we assume $P(\langle c, \sigma[0/x]\rangle \to \sigma') \overset{def}{=} \mathcal{C}[\![c]\!](\sigma[0/x]) = \sigma'$

we want to prove $P(\langle \text{reset } x \text{ in } c, \sigma\rangle \to \sigma'[\sigma x/x]) \overset{def}{=} \mathcal{C}[\![\text{reset } x \text{ in } c]\!]\sigma = \sigma'[\sigma x/x]$

$\mathcal{C}[\![\text{reset } x \text{ in } c]\!]\sigma = \left(\mathcal{C}[\![c]\!](\sigma[0/x])\right)[\sigma x/x] = \sigma'[\sigma x/x]$

(1.3) b) $P(c) \overset{def}{=} \forall \sigma \sigma'.\ \mathcal{C}[\![c]\!]\sigma = \sigma' \Rightarrow \langle c, \sigma\rangle \to \sigma'$ by structural induction

we want to prove $P(\text{reset } x \text{ in } c) \overset{def}{=} \forall \sigma \sigma'.\ \mathcal{C}[\![\text{reset } x \text{ in } c]\!]\sigma = \sigma' \Rightarrow \langle \text{reset } x \text{ in } c, \sigma\rangle \to \sigma'$

we assume $\mathcal{C}[\![\text{reset } x \text{ in } c]\!]\sigma = \sigma'$ we want to prove $\langle \text{reset } x \text{ in } c, \sigma\rangle \to \sigma'$

$$\left(\mathcal{C}[\![c]\!](\sigma[0/x])\right)[\sigma x/x] = \sigma'$$

Let $\sigma'' = \mathcal{C}[\![c]\!]\sigma[0/x]$ by structural ind. hypothesis we know

$$P(c) = \mathcal{C}[\![c]\!]\sigma[0/x] = \sigma'' \Rightarrow \langle c, \sigma[0/x]\rangle \to \sigma''$$

by applying the rule we conclude $\langle \text{reset } x \text{ in } c, \sigma\rangle \to \sigma''[\sigma x/x] = \sigma'$